

TESTING FOR HETEROSCEDASTICITY AND SERIAL CORRELATION IN A TWO-WAY ERROR COMPONENT MODEL

BY

Sam O.OLOFIN¹, Eugene KOUASSI², and Afees Adebare SALISU³

1.0 Introduction

1.1 Statement of the Problem

It is increasingly becoming a common practice in empirical literature of macroeconomics to use cross-country samples for econometric analyses. Part of the explanations is paucity of time series data available for adequate analysis, especially for the less developed countries. A more general explanation is the claim that cross-country data at least allow one to identify aggregate relationships or correlations in the data that appear to hold ‘on average’ over a wide sample. The weakness of this approach, however, is that what appears to hold on average is rarely an adequate explanation of what is happening in a particular country. Furthermore, the macroeconomic relationships of interest are typically dynamic in nature, so that researchers should be interested in what is happening in countries over time (see Lloyd, Morrisey and, Osei, 2001). Cross-section techniques are limited in their ability to address these concerns.

Alternatively, panel data techniques are employed to capture time series dimension and/or ‘smooth out’ year-on-year variability in the data. These data allow us to construct and test more realistic behavioural models that could not be identified using either a cross section or a single time series data set. Panel data models have become increasingly popular among applied researchers due to their heightened capacity for

¹ **Sam O.OLOFIN** is Professor of Economics and Director, Centre for Econometric and Allied Research (CEAR), Department of Economics, University of Ibadan, Nigeria. E-mail: soolofin@yahoo.com

² **Eugene KOUASSI** is Resource Economics – West Virginia University (USA). E-mail: Eugene.kouassi@gmail.com

³ **Afees Adebare SALISU** is a Doctoral Candidate and Junior Research Fellow, Centre for Econometric and Allied Research (CEAR), Department of Economics, University of Ibadan, Ibadan, Nigeria. E-mail: adebare1@yahoo.com. Phone No.: +2348034711769.

capturing the complexity of human behaviour unlike cross-sectional or time series data models (see Hsiao, 2003). Despite these, panel data are subject to their own experimental problems. Prominent among the problems constantly addressed in panel data econometrics are selectivity and heterogeneity biases. According to Hsiao (2003), *'it is only by taking proper account of selectivity and heterogeneity biases in the panel data that one can have confidence in the results obtained.'* Often, however, researchers ignore the existence of these problems when carrying out panel data analysis. This is not unconnected with the fact that a standard panel data model assumes that regression disturbances are homoscedastic with the same variance across time and individuals (thereby ignoring the possibility of selectivity and heterogeneity biases). This assumption may be a restrictive one for panels, where the cross sectional units may be of varying sizes and as a result may exhibit different variations. Although, assuming homoscedastic disturbances when heteroscedasticity is present will still result in consistent estimates of the regression coefficients, these estimates will not be efficient (see Baltagi, 2008). Similarly, error component disturbances in the random effect model assume that the only correlation over time is due to the presence of the same individual across the panel (also ignoring the possibility of heterogeneity biases). This may also be a restrictive assumption for economic relationships like investment or consumption, where unobserved shock in this period will affect the behavioral relationship for at least the next few periods. However, this type of serial correlation is not taken into account in the error component model. Yet, ignoring serial correlation when it is present results in consistent but inefficient estimates of regression coefficients and biased standard errors (see Baltagi, 2005).

These are essentially the estimation problems that both theoretical and applied econometricians have constantly been tackling in panel econometrics. On one side are those who look at heteroscedasticity problem only while ignoring the possibility of serial correlation⁴. On the other side are those who consider serial correlation only while

⁴ Prominent among these papers include Mazodier and Trognon (1978) Rao et al. (1981), Magnus (1982), Baltagi (1988), Baltagi and Griffin (1988), Randolph (1988), Wansbeek (1989), Li and Stengos (1994), Holly and Gardiol (2000), Roy (2002), Phillips (2003) and Baltagi, Bresson and Pirotte (2006)

ignoring the possibility of heteroscedasticity.⁵ The potential problem with theoretical derivations emerging from these studies is that although allowing for homoscedasticity only when in fact serial correlation also exists will still yield consistent estimates of regression coefficients, but these will be inefficient and of biased standard errors. Similarly, testing for serial correlation only when in fact heteroscedasticity also exists will produce the same effects. The implication of this problem is that one may be accepting what should have been rejected and in a situation where policies are based on such econometric analyses, policy/decision makers would have been misled and the public (both individual households and the investors) will feel the consequential effect. Perhaps because of this, Baltagi, Jung and Song (2008) considered a panel data regression model with heteroscedastic as well as serially correlated disturbances, and consequently derived a joint Lagrangian Multiplier (LM) test for homoscedasticity and no first order serial correlation, a conditional LM test for homoscedasticity given serial correlation, and a conditional LM test for no serial correlation given heteroscedasticity. They extend the work of Holly and Gardiol (2000) who consider only heteroscedasticity in a one-way error component model. While the Baltagi et. al. (2008) paper seems to address the problems inherent in the previous literature; their analysis was based on the one-way error component model. However, and particularly in the microeconomic literature, there are some factors that are specific to time periods which influence significantly the behaviour of firms and individual households (time-specific effects) which the work of Baltagi et. al. (2008) did not consider. Therefore, their derivations and propositions are limited to individual specific effects. The present study intends to overcome this limitation. We intend to extend the works of Holly and Gardiol (2000) and Baltagi et al. (2008) by testing for both heteroscedasticity and serial correlation in a two-way error components model, which takes care of both individual- and time-specific effects.

⁵ Among the notable works on the problem of serial correlation in panel data include Lillard and Willis (1978), Bhargava, Franzini, and Narendranathan (1982) Burke, Godfrey and Termayne (1990), Baltagi and Li (1991, 1994, 1995) Galbraith and Zinde-Wash (1992, 1995)

1.2 Objectives of the Study

As pointed out, the main objective of this work is to extend and generalize Holly and Gardiol (2000) and Baltagi et al. (2008) works to allow for serial correlation in the time parameter as well as in the remainder disturbances in the context of the two-way error component model. Specifically, the study intends to

- (i) derive a joint LM test for homoscedasticity when no first order serial correlation is present;
- (ii) derive a conditional LM test for homoscedasticity when serial correlation is present;
- (iii) derive a conditional LM test for no first order serial correlation given heteroscedasticity; and
- (iv) examine the power and size of the above tests under various forms of heteroscedasticity and autocorrelation.

1.3 Justification for the Study

Evidently, traditional error components model has been extended to take into account serial correlation (see Lillard and Willis (1978), Baltagi and Li (1995), Galbraith and Zinde-Walsh (1995), Bera, Sosa-Escudero and Yoon (2001) and Hong and Kao (2004)). Equally, the model has been generalized to take into account heteroscedasticity (see Mazodier and Trognon (1978), Baltagi and Griffin (1988), Li and Stengos (1994), Lejeune (1996), Holly and Gardiol (2000), Roy (2002), Baltagi, Bresson and Pirotte (2006) and Kouassi and Kymn (2008)). However, one noticeable gap in the literature is that theoretical econometricians are separated in terms of their treatment of panel data error components. One side are those who believe that the prominent problem in the error components model is heteroscedasticity and as such focus on this problem. On the other, are those who conclude the prominent problem is serial correlation while ignoring the problem of heteroscedasticity (see Baltagi, 2008). Baltagi, Bresson and Pirotte (2006) for example, derived a Lagrange Multiplier (LM) test which jointly tests for the presence of serial correlation as well as random individual effects assuming homoscedasticity of the disturbances. Holly and Gardiol (2000), however, derived an LM statistic which tests for

homoscedasticity of the disturbances while no serial correlation in the remainder disturbances in the context of a one way random effects panel data model. As earlier emphasized, allowing for homoscedasticity only when in fact serial correlation also exists will still yield consistent estimates of regression coefficients, but these will be inefficient and of biased standard errors. Similarly, testing for serial correlation only when in fact heteroscedasticity also exists will produce the same effects. The implication of this is that we may be accepting research hypotheses that are supposed to be rejected and vice-versa.

Although, Baltagi et. al. (2008) tried to resolve this problem by testing for both heteroscedasticity and serial correlation, they however, ignored the time parameter (an essential parameter in most cross-section analyses). Our intention in this study is to fill this research gap by accounting for both individual and time-specific effects in a panel data model. This type of panel data model is called the two-way error component model and as emphasized earlier, we will derive a joint LM test for homoscedasticity and no first order serial correlation, a conditional LM test for homoscedasticity given serial correlation and a conditional LM test for no first order serial correlation given heteroscedasticity. The benefit of this exercise is not only limited to its contribution to the existing literature on panel econometrics, it will also give researchers some level of confidence in their results when panel data models are considered for analysis.

1.4 Research Hypotheses

Given the following two-way error component model:

$$y_{it} = x_{it}\beta + u_{it} ; i=1, \dots, N, t=1, \dots, T, \quad (1)$$

$$u_{it} = \mu_i + \lambda_t + v_{it} \quad (2)$$

Where i indexes individual units up to the N individuals and t indexes time periods up to the T periods of each individual i . y_{it} represents the observation on the dependent variable for the i th individual at the t th time period, x_{it} denotes the $k \times 1$ vector of observations on the nonstochastic regressors, the testable research hypotheses are

First Group of Tests

$$H_0 : \sigma_{\mu_i}^2 = \sigma_{\mu}^2, \forall i \text{ and } \sigma_{\lambda_i}^2 = 0 \text{ but } \sigma_{v_{it}}^2 \neq 0$$

$$H_0 : \sigma_{\mu_i}^2 = \sigma_{\mu}^2, \forall i \text{ and } \sigma_{\lambda_i}^2 \neq 0 \text{ but } \sigma_{v_{it}}^2 = 0$$

$$H_0 : \sigma_{\mu_i}^2 = \sigma_{\mu}^2, \forall i \text{ and } \sigma_{\lambda_i}^2 = 0 \text{ and } \sigma_{v_{it}}^2 = 0$$

In the first group of tests, we examine homoscedasticity under various assumptions.

Second Group of Tests

$$H_0 : \sigma_{\mu_i}^2 = \sigma_{\mu}^2, \forall i \text{ given } \sigma_{\lambda_i}^2$$

$$H_0 : \sigma_{\mu_i}^2 = \sigma_{\mu}^2, \forall i \text{ given } \sigma_{v_{it}}^2$$

$$H_0 : \sigma_{\mu_i}^2 = \sigma_{\mu}^2, \forall i \text{ given } \sigma_{\lambda_i}^2 \text{ and } \sigma_{v_{it}}^2$$

The second group of tests still examine homoscedasticity but given various assumptions.

Third Group of Tests

$$H_0 : \sigma_{\mu_i}^2 \neq \sigma_{\mu}^2, \forall i \text{ and } \sigma_{\lambda_i}^2 = 0 \text{ but } \sigma_{v_{it}}^2 \neq 0$$

$$H_0 : \sigma_{\mu_i}^2 \neq \sigma_{\mu}^2, \forall i \text{ and } \sigma_{\lambda_i}^2 \neq 0 \text{ but } \sigma_{v_{it}}^2 = 0$$

$$H_0 : \sigma_{\mu_i}^2 \neq \sigma_{\mu}^2, \forall i \text{ and } \sigma_{\lambda_i}^2 = 0 \text{ and } \sigma_{v_{it}}^2 = 0$$

The third group of tests deals with heteroscedasticity under various assumptions.

Fourth Group of Tests

$$H_0 : \sigma_{\mu_i}^2 \neq \sigma_{\mu}^2, \forall i \text{ given } \sigma_{\lambda_i}^2$$

$$H_0 : \sigma_{\mu_i}^2 \neq \sigma_{\mu}^2, \forall i \text{ given } \sigma_{v_{it}}^2$$

$$H_0 : \sigma_{\mu_i}^2 \neq \sigma_{\mu}^2, \forall i \text{ given } \sigma_{\lambda_i}^2 \text{ and } \sigma_{v_{it}}^2$$

The last group of tests is concerned with heteroscedasticity given various assumptions.

1.5 Scope of the Study

The focus of this study is on Panel Econometrics with specific interest in the two-way error component model. Essentially, the work looks at the twin problems of heteroscedasticity and serial correlation in the two-way error component model. The

choice of this area of panel econometrics is informed by the gap in the literature in which there is no study that has looked into the joint occurrence of heteroscedasticity and serial correlation in a two-way error component model.

2.0 Review of Related Literature

Mazodier and Trognon (1978) seem to be the first to deal with the problem of heteroscedasticity in panel data. The study looks at heteroscedasticity and stratification in two-way error component models. The study involves spectral decomposition of the variance-covariance matrix to derive statistically efficient and computationally simple estimation procedures. Although, this paper considers the two-way error component model, it only accounts for heteroscedasticity and ignores the possibility of serial correlation problem in the model. The present study, however, considers both.

The pioneering work of Mazodier and Trognon (1978) has given rise to further studies. Prominent among these papers are Rao et al. (1981), Magnus (1982), Baltagi (1988), Baltagi and Griffin (1988), Randolph (1988), Wansbeek (1989), Li and Stengos (1994), Holly and Gardiol (2000), Roy (2002), Phillips (2003) and Baltagi, Bresson and Pirote (2006) . However, these works were concerned with regression models with one-way error components disturbances: $u_{it} = \mu_i + v_{it}$, $i = 1, \dots, T$, where the index i refers to the T time series observations. For example, both Mazodier and Trognon (1978) and Baltagi and Griffin (1988) were concerned with the estimation of a model allowing for heteroscedasticity on the individual-specific error term, i.e., assuming that $\mu_i \sim (0, \sigma_{\mu_i}^2)$ while $v_{it} \sim IID(0, \sigma_v^2)$. In contrast, Rao et al. (1981), Magnus (1982), Baltagi (1988) and Wansbeek (1989) adopted a symmetrically opposite specification allowing for heteroscedasticity on the remainder error term, i.e., assuming that $\mu_i \sim IID(0, \sigma_{\mu}^2)$ while $v_{it} \sim (0, \sigma_{v_i}^2)$. Randolph (1988) allowed for a more general heteroscedastic error component model assuming that both the individual and remainder error terms were heteroscedastic, i.e., $\mu_i \sim (0, \sigma_{\mu_i}^2)$ and $v_{it} \sim (0, \sigma_{v_{it}}^2)$, with the latter varying with every observation over time and individuals.

Lejeune (1996) on the other hand, dealt with Maximum Likelihood Estimation (MLE) and LM testing of a general heteroscedastic one-way error components regression model which assumes that $\mu_i \sim (0, \sigma_{\mu_i}^2)$ and $v_{it} \sim (0, \sigma_{v_{it}}^2)$, where $\sigma_{\mu_i}^2$ and $\sigma_{v_{it}}^2$ are distinct parametric functions of exogenous variables z_{it}' and f_i' , i.e., $\sigma_{v_{it}}^2 = \sigma_v^2 h_v(z_{it}'\theta_1)$ and $\sigma_{\mu_i}^2 = \sigma_{\mu}^2 h_{\mu}(f_i'\theta_2)$. In the context of incomplete panels, Lejeune (1996) derives two joint LM tests for no individual effects and homoscedasticity in the remainder error term. In the first LM test, he considers a random-effects-one-way error component model with $\mu_i \sim iidN(0, \sigma_{\mu}^2)$ and a remainder error term that is heteroscedastic $v_{it} \sim N(0, \sigma_{v_{it}}^2)$ with $\sigma_{v_{it}}^2 = \sigma_v^2 h_v(z_{it}'\theta_1)$. The joint hypothesis $\theta_1 = \sigma_{\mu}^2 = 0$ renders OLS the restricted MLE. Lejeune argues that there is no need to consider a variance function for μ_i since one is testing $\sigma_{\mu}^2 = 0$. While the computation of the LM test statistic is simplified under this assumption, i.e., $\mu_i \sim iidN(0, \sigma_{\mu}^2)$, this is not in the original spirit of Lejeune's ML estimation where both μ_i and v_{it} have general variance functions. Lejeune's second LM test considers a fixed effects one-way error component model, where μ_i is a fixed parameter to be estimated and the remainder error term is heteroscedastic with $v_{it} \sim N(0, \sigma_{v_{it}}^2)$ and $\sigma_{v_{it}}^2 = \sigma_v^2 h_v(z_{it}'\theta_1)$. The joint hypothesis is $\mu_i = \theta_1 = 0$ for all $i = 1, 2, \dots, N$. This renders OLS the restricted MLE.

With regards to estimation, Li and Stengos (1994) suggested an adaptive estimation procedure for an error component model allowing for heteroscedasticity of unknown form on the remainder error term, i.e., assuming that $\mu_i \sim IID(0, \sigma_{\mu}^2)$ while $v_{it} \sim (0, \sigma_{v_{it}}^2)$, where $\sigma_{v_{it}}^2$ is a nonparametric function $f(z_{it}')$ of a vector of exogenous variables. They also suggest a robust version of the Breusch and Pagan (1980) LM test for no random individual effects, $\sigma_{\mu}^2 = 0$, by allowing for adaptive heteroscedasticity of unknown form on the remainder error term. Holly and Gardiol (2000) propose a Rao score test for homoscedasticity assuming the existence of individual effects. The unrestricted model assumes that $\mu_i \sim N(0, \sigma_{\mu_i}^2)$ and $v_{it} \sim iidN(0, \sigma_v^2)$ where $\sigma_{\mu_i}^2$ is a parametric function of exogenous variables f_i' , i.e., $\sigma_{\mu_i}^2 = \sigma_{\mu}^2 h_{\mu}(f_i'\theta_2)$. Under the null

hypothesis $\theta_2 = 0$, with $h_\mu(0) = 1$, the restricted model reverts to the homoscedastic one-way error component model. Roy (2002) deals with adaptive estimation of an error component model assuming heteroscedasticity of unknown form for individual-specific error terms, i.e., assuming that $\mu_i \sim (0, \sigma_{\mu_i}^2)$ while $v_{it} \sim IID(0, \sigma_v^2)$, where $\sigma_{\mu_i}^2$ is a nonparametric function $f(\bar{z}'_i)$ of a vector of individual means of exogenous variables z'_{it} with $\bar{z}'_i = \sum_{t=1}^T z'_{it} / T$. More recently, Phillips (2003) follows Mazodier and Trognon (1978) in considering a one-way stratified error component model. As unobserved heterogeneity occurs through individual-specific variances changing across strata, Phillips provides an algorithm for estimating this model and suggests a bootstrap test for identifying the number of strata.

In relation to the general heteroscedastic model of Randolph (1988) and Lejeune (1996), Baltagi, Bresson and Pirotte (2006) derive a joint LM test for homoscedasticity, i.e., $\theta_1 = \theta_2 = 0$. Under the null hypothesis, the model is a homoscedastic one-way error component regression model and is estimated by restricted MLE. This is different from Lejeune (1996), where under the null, $\sigma_\mu^2 = 0$, so that the restricted MLE is OLS and not MLE on a one-way homoscedastic error component model. This model under the null is exactly that of Holly and Gardiol (2000) but it is more general under the alternative since it does not assume a homoscedastic remainder error term. Baltagi, Bresson and Pirotte (2006) also derive an LM test for the null hypothesis of homoscedasticity of the individual random effects assuming homoscedasticity of the remainder error term, i.e., $\theta_2 = 0 | \theta_1 = 0$. Their LM test is not different from Holly and Gardiol (2000). In addition, they derive an LM test for the null hypothesis of homoscedasticity of the remainder error term assuming homoscedasticity of the individual effects, i.e., $\theta_1 = 0 | \theta_2 = 0$.⁶

Similarly, among the notable works on the problem of serial correlation in panel data are Lillard and Willis (1978), Bhargava, Franzini, and Narendranathan (1982) Burke, Godfrey and Termayne (1990), Baltagi and Li (1991, 1994, 1995) Galbraith and

⁶ The review of related literature on heteroscedasticity in a panel data model was extensively drawn from Baltagi, Bresson and Pirotte (2006).

Zinde-Wash (1992, 1995). The error component model was extended to take into account first-order serial correlation in the remainder disturbances by Lillard and Willis (1978) for the random effects model and by Bhargava, Franzini, and Narendranathan (1982) for the fixed effects model. Both studies considered the first order Autoregressive [AR(1)] specification on the remainder disturbances. Nicholls, Pagan, and Terrell (1975), while considering first order moving average [MA(1)], find MA(1) is a viable alternative to AR(1). Baltagi and Li (1991) give a transformation which may be applied to certain autocorrelated disturbances in an error components model to yield spherical disturbances. They derive the transformations for first order Autoregressive [AR(1)] and second order Autoregressive [AR(2)] cases.

In furtherance to this theoretical paper, Baltagi and Li (1994) provide a simple estimation method for an error component regression model with q th order Moving Average [MA(q)] remainder disturbances. Their estimation method utilizes the transformation derived by Baltagi and Li (1991) for an error component model with autoregressive remainder disturbances, and a standard orthogonalizing algorithm for the general MA(q) model. Comprehensively, Baltagi and Li (1995) combine their earlier works in 1991 and 1994 by testing AR(1) against MA(1) disturbances in an error component model. The authors derive three Lagrangian Multiplier (LM) statistics for an error component model with first-order serially correlated errors. The first LM statistic jointly tests for zero first-order serial correlation and random individual effects, second LM statistic tests for zero first-order serial correlation assuming fixed individual effects, and the third LM statistic tests for zero first-order serial correlation assuming random individual effects. In all the three cases, the authors find that the corresponding LM statistic is the same whether the alternative is AR(1) or MA(1). The study also derives two extensions of the Burke, Godfrey and Termayne (1990) test from the time-series to the Panel data literature. The first tests the null of AR(1) disturbances against MA(1) disturbances, and the second tests the null of MA(1) disturbances against AR(1) disturbances in an error component model. The tests are computationally simple requiring only OLS or within residuals.

Galbraith and Zinde-Walsh (1995) also considered orthogonalizing transformation for the error-component model with serially correlated disturbances in the

general ARMA case. Their work involves generalizations of results obtained by the Baltagi and Li (1991) study of the AR (1) or AR (2) error processes.

It is evident from the foregoing that the strands of literature on serial correlation and heteroscedasticity problem in panel data analysis are almost separate. When the focus is on heteroscedasticity, serial correlation is ignored, and when the focus is on serial correlation, heteroscedasticity is ignored. The only notable work that assumes the existence of both serial correlation and heteroscedasticity problems is Baltagi, Jung and Song (2008).

Baltagi, Jung and Song (2008) consider a panel data regression model with heteroscedastic as well as serially correlated disturbances, and derive a joint LM test for homoscedasticity and no first order serial correlation. The restricted model is the standard random individual error component model. They also derive a conditional LM test for homoscedasticity given serial correlation and a conditional LM test when there is no first order serial correlation given heteroscedasticity, all in the context of a random effects panel data model. Monte Carlo results show that these tests along with their likelihood ratio alternatives have good size and power under various forms of heteroscedasticity. One of the fundamental differences between Baltagi, Jung and Song (2008) and the present work is that our work considers a more comprehensive model – the two-way error components model that takes care of both random individual and time effects- to test for the probable existence of heteroscedasticity and serial correlation in the model. The present study is, therefore, broader in scope.

Table 1: A summary of evidence from previous studies and the intended contribution of the present research work

S/N	Research Area	Authors	Nature of Panel Data Model	Prominent Contributions	Intended Contribution of the Present Study to the Literature
1.	Testing for Heteroscedasticity in a Panel Data model	Mazodier and Trognon (1978), Rao et al. (1981), Magnus (1982), Baltagi (1988), Baltagi and Griffin (1988), Randolph (1988), Wansbeek (1989), Li and Stengos (1994), Holly and Gardiol (2000), Roy (2002) and Phillips (2003), Baltagi, Bresson and Pirotte (2006) .	One-Way Error Component Model	Derivation of LM test	To consider a two-way error component model and to derive a conditional LM test for heteroscedasticity when there is no serial correlation.
2.	Testing for Serial Correlation in a Panel Data model	Lillard and Willis (1978), Bhargava, Franzini, and Narendranathan (1982) Burke, Godfrey and Termayne (1990), Baltagi and Li (1991, 1994, 1995) Galbraith and Zinde-Wash (1992, 1995) and Hong and Kao	One-Way Error Component Model	Derivation of LM test	To consider a two-way error component model and to derive a conditional LM test for homoscedasticity given serial correlation.

		(2004).			
3.	Testing for Both Serial Correlation and Heteroscedasticity in a Panel Data model	Baltagi, Jung and Song (2008)	One-Way Error Component	<p>(1) Derivation of a joint LM test for homoscedasticity and no first serial correlation.</p> <p>(2) Derivation of a conditional LM test for homoscedasticity given serial correlation.</p> <p>(3) Derivation of a conditional LM test for no serial correlation given heteroscedasticity.</p>	<p>To consider a two-way error component model in the derivation of the following:</p> <p>(1) A joint LM test for homoscedasticity and no first order serial correlation.</p> <p>(2) A conditional LM test for homoscedasticity given serial correlation.</p> <p>(3) A conditional LM test for no serial correlation given heteroscedasticity.</p>

3.0 Theoretical Framework and Methodology

3.1 Theoretical Framework

Essentially, panel data analysis considers either one-way error components model or the two-way error components model or both. The one-way error component model accounts for individual specific effects but it ignores the time-specific effect while the two-way error component model accounts for both. As evidenced in the theoretical literature on panel econometrics, the analysis of the problem of heteroscedasticity and serial correlation has been in relation to one-way error components model. The present research work intends to fill a research gap by considering this problem in the context of the two-way error component model. However, our analysis will be guided by the existing literature on the one-way error component model.

Consider the following two-way error component model:

$$y_{it} = x_{it}\beta + u_{it} ; \quad i=1, \dots, N, \quad t=1, \dots, T, \quad (1)$$

Where i indexes individual units up to the N individuals and t indexes time periods up to the T periods of each individual i . y_{it} represents the observation on the dependent variable for the i th individual at the t th time period, x_{it} denotes the $k \times 1$ vector of observations on the nonstochastic regressors. In the context of a two-way error component model, the regression disturbance term is described by the equation:

$$u_{it} = \mu_i + \lambda_t + v_{it} \quad (2)$$

With μ_i representing individual-specific effect, λ_t , time-specific effect and v_{it} , the remainder disturbance term.

The stochastic error components have the following properties assuming homoscedasticity and no serial correlation in the error components:

$$\mu_i \sim IID(0, \sigma_\mu^2); \quad \lambda_t \sim IID(0, \sigma_\lambda^2); \quad \text{and} \quad v_{it} \sim IID(0, \sigma_v^2) \quad (3)$$

$$E(\mu_i v_{js}) = 0; \quad E(\lambda_t v_{js}) = 0 \quad \forall i, j, s, \quad \text{and} \quad E(v_{is} v_{jt}) = 0 \quad \forall s, t, \quad i \neq j; \quad (4)$$

By relaxing these restrictive assumptions on μ_i , λ_t and v_{it} in the context of a two-way error components model, we have the following variations:

- (a) A case of homoscedastic random individual and time effects while allowing for heteroscedastic remainder disturbance term. Under this assumption, we have the following properties for the error components:

$$\mu_i \sim IID(0, \sigma_\mu^2); \lambda_t \sim IID(0, \sigma_\lambda^2) \text{ and } v_{it} \sim (0, \sigma_{v_{it}}^2) \quad (5)$$

- (b) A case of homoscedastic random individual while allowing for heteroscedastic time effects and remainder disturbance term. Under this assumption, we have the following properties for the error components:

$$\mu_i \sim IID(0, \sigma_\mu^2); \lambda_t \sim (0, \sigma_{\lambda_t}^2) \text{ and } v_{it} \sim (0, \sigma_{v_{it}}^2) \quad (6)$$

- (c) A case of homoscedastic random individual and remainder disturbance term while allowing for heteroscedastic time effects. Under this assumption, we have the following properties for the error components:

$$\mu_i \sim IID(0, \sigma_\mu^2); \lambda_t \sim (0, \sigma_{\lambda_t}^2) \text{ and } v_{it} \sim IID(0, \sigma_v^2) \quad (7)$$

- (d) A case of homoscedastic remainder disturbance term while allowing for heteroscedastic individual and time effects. Here,

$$\mu_i \sim (0, \sigma_{\mu_i}^2); \lambda_t \sim (0, \sigma_{\lambda_t}^2) \text{ and } v_{it} \sim IID(0, \sigma_v^2) \quad (8)$$

- (e) A case of homoscedastic time effect and remainder disturbance term while allowing for heteroscedastic individual effects. Here,

$$\mu_i \sim (0, \sigma_{\mu_i}^2); \lambda_t \sim IID(0, \sigma_\lambda^2) \text{ and } v_{it} \sim (0, \sigma_v^2) \quad (9)$$

- (f) A case of homoscedastic time effects while allowing for heteroscedastic random individual and remainder disturbance term. Under this assumption, we have the following properties for the error components:

$$\mu_i \sim (0, \sigma_{\mu_i}^2); \lambda_t \sim IID(0, \sigma_\lambda^2) \text{ and } v_{it} \sim (0, \sigma_{v_{it}}^2) \quad (10)$$

- (g) A case of heteroscedasticity in all the error components This implies that:

$$\mu_i \sim (0, \sigma_{\mu_i}^2); \lambda_t \sim (0, \sigma_{\lambda_t}^2) \text{ and } v_{it} \sim (0, \sigma_{v_{it}}^2) \quad (11)$$

- (h) A case of homoscedastic random individual and time effects when first order serial correlation of the remainder disturbance term is present. In this case, $\mu_i \sim IID(0, \sigma_\mu^2); \lambda_t \sim IID(0, \sigma_\lambda^2)$ and the remainder disturbances can follow an

AR(1) process i.e., $v_{it} = \rho v_{i,t-1} + \varepsilon_{it}$ with $|\rho| < 1$, or an MA (1) process i.e., $v_{it} = \varepsilon_{it} + \lambda \varepsilon_{i,t-1}$ with $|\lambda| < 1$. In both cases, we assume $\varepsilon_{it} \sim IIN(0, \sigma_\varepsilon^2)$.

- (i) A case of heteroscedastic random individual and time effects in the absence of first order serial correlation of the remainder disturbance term. Here, $\mu_i \sim (0, \sigma_{\mu_i}^2)$ and $\lambda_t \sim (0, \sigma_{\lambda_t}^2)$ and since there is no first order serial correlation in the remainder disturbance term, $|\rho|$ and $|\lambda|$ equal to zero and, therefore, $v_{it} = \varepsilon_{it}$ for both an AR (1) and an MA (1) processes. Given that ε_{it} is white noise, therefore, v_{it} is homoscedastic i.e., $v_{it} \sim IID(0, \sigma_v^2)$.
- (j) A case of homoscedastic random individual but heteroscedastic time effects when first order serial correlation of the remainder disturbance term is present. In this case, $\mu_i \sim IID(0, \sigma_\mu^2)$; $\lambda_t \sim (0, \sigma_{\lambda_t}^2)$ and the remainder disturbances can follow an AR (1) process i.e., $v_{it} = \rho v_{i,t-1} + \varepsilon_{it}$ with $|\rho| < 1$, or an MA (1) process i.e., $v_{it} = \varepsilon_{it} + \lambda \varepsilon_{i,t-1}$ with $|\lambda| < 1$. In both cases, we assume $\varepsilon_{it} \sim IIN(0, \sigma_\varepsilon^2)$. By using the lag operator: for an AR(1) process, we have $v_{i,0} \sim IIN(0, \sigma_\varepsilon^2 / (1 - \rho^2))$; for an MA(1) process, we have $v_{i,0} \sim (0, \sigma_\varepsilon^2 (1 + \lambda^2))$.
- (k) A case of homoscedastic time effects allowing for heteroscedastic random individual in the absence of first order serial correlation of the remainder disturbance term.
- (l) A case of homoscedastic time effects allowing for heteroscedastic random individual in the presence of first order serial correlation of the remainder disturbance term.
- (m) A case of homoscedastic random individual allowing for time effects in the absence of first order serial correlation of the remainder disturbance term.
- (n) A case of heteroscedastic random individual and time effects when first order serial correlation of the remainder disturbance term is also present.

Note that we did not bother to define the distributions for μ_i, λ_t and v_{it} for cases labeled k, l, m and n because we have similar ones already defined in the previous cases

These variations are also presented in table 2 below.

Table 2: Possible joint occurrence of heteroscedasticity and serial correlation using the two-way error component model

Individual Specific Effect	Time Specific Effect	Remainder Disturbance term
Homoscedasticity	Homoscedasticity	First order serial correlation
Homoscedasticity	Heteroscedasticity	First order serial correlation
Heteroscedasticity	Homoscedasticity	First order Serial Correlation
Heteroscedasticity	Heteroscedasticity	First order Serial Correlation
Heteroscedasticity	Heteroscedasticity	No first order serial correlation
Homoscedasticity	Heteroscedasticity	No first order serial correlation
Heteroscedasticity	Homoscedasticity	No first order serial correlation
Homoscedasticity	Homoscedasticity	Heteroscedasticity
Homoscedasticity	Heteroscedasticity	Heteroscedasticity
Heteroscedasticity	Homoscedasticity	Heteroscedasticity
Heteroscedasticity	Homoscedasticity	Homoscedasticity
Homoscedasticity	Heteroscedasticity	Homoscedasticity
Heteroscedasticity	Heteroscedasticity	Homoscedasticity
Heteroscedasticity	Heteroscedasticity	Heteroscedasticity

Source: Compiled by the Author

Expressed above are the possible econometric problems that a researcher is likely to encounter using a two-way error component model. However, there is no way any of these problems can be tested as there is no available technique in the literature for carrying out such tests. This is a gap the present work intends to fill. We will develop LM tests under the various assumptions and consequently we will generate data to validate the tests.

3.2 Methodology

The methodology of this work is based on a generalization of the one used in Baltagi et al. (2008). Essentially, we shall consider the two-way error component model in testing for heteroscedasticity and serial correlation in panel data which is an extension to the existing theoretical literature on panel data. The following steps are of interest,

STEP 1: Write the variance-covariance matrix of the error term;

STEP 2: Write the variance-covariance matrix of the error term in the presence of AR errors;

STEP 3: Consider the variance-covariance matrix of the transformed model;

STEP 4: Obtain the spectral decomposition of the above matrix using the method of Wansbeek and Kapteyn (1982) and Baltagi and Li (1995);

STEP 5: Obtain the inverse of the transformed variance covariance matrix based on the method developed by Magnus (1982).

STEP 6: Derive the log – likelihood functions;

STEP 7: Using general formulas on log-likelihood differentiation (e.g., see Hemmerle and Hartley, 1973 and Harville, 1977) derive the score functions of the likelihood evaluated from the restricted and unrestricted MLE under the null;

STEP 8: Using the results of Harville (1977), derive the information matrix;

STEP 9: Derive the inverse of the information matrix;

STEP 10: Derive the LM tests under various assumptions.

3.3 Monte Carlo Data Generating Process

The Monte Carlo simulations method will be used to generate data necessary for testing the suitability and reliability of the proposed extensions to panel econometrics. The design of Monte Carlo experiments will follow closely that of Baltagi, Jung and Song (2008), Baltagi, Bresson and Pirotte (2006) and Li and Stengos (1994). The only difference, however, is the inclusion of time parameter. Consider a simple two-way error components model (see equation 1):

$$y_{it} = \beta_0 + \beta_1 x_{it} + \mu_i + \lambda_t + v_{it}, \quad i = 1, \dots, N; t = 1, \dots, T. \quad (12)$$

Where

$$x_{it} = w_{i,t} + 0.5w_{i,t-1} \quad (13)$$

$w_{i,t}$ is uniformly distributed on the interval $[0, 2]$. The parameters β_0 and β_1 are assigned 5 and 0.5 respectively. We will then choose $N = 50, 100$ and 200 and $T = 5$ and 10 . For each x_{it} , we generate $T+10$ observations and drop the first 10 observations in order to reduce the dependency on initial values. In addition, we will assume that for serial correlation, v_{it} follows an AR(1) process i.e., $v_{it} = \rho v_{i,t-1} + \varepsilon_{it}$ with $\varepsilon_{it} \sim IIN(0, \sigma_\varepsilon^2)$. The initial values $v_{i,0}$ will be generated as $v_{i,0} \sim IIN(0, \sigma_\varepsilon^2 / (1 - \rho^2))$. The autocorrelation coefficient ρ varies over the set 0 to 0.5 by increments of 0.1.

For the individual and time effects heteroscedasticity, we will modify the Roy (2002) set up to account for the time parameter. Specifically, we will generate

$\mu_i \sim (0, \sigma_{\mu_i}^2)$; $\lambda_t \sim (0, \sigma_{\lambda_t}^2)$ and $\varepsilon_{it} \sim (0, \sigma_\varepsilon^2)$ where

$$\sigma_{\mu_i}^2 = \sigma_{\mu_i}^2 (\bar{x}_i) = \sigma_{\mu_i}^2 (1 + \alpha_{\mu_i} \bar{x}_i)^2 \quad (14)$$

$$\sigma_{\lambda_t}^2 = \sigma_{\lambda_t}^2 (\bar{x}_t) = \sigma_{\lambda_t}^2 (1 + \alpha_{\lambda_t} \bar{x}_t)^2 \quad (15)$$

Equations (14) and (15) are denoted as quadratic heteroscedasticity. For exponential heteroscedasticity, we have:

$$\sigma_{\mu_i}^2 = \sigma_{\mu_i}^2 (\bar{x}_i) = \sigma_{\mu_i}^2 \exp(\alpha_{\mu_i} \bar{x}_i); \quad (16)$$

$$\sigma_{\lambda_t}^2 = \sigma_{\lambda_t}^2 (\bar{x}_t) = \sigma_{\lambda_t}^2 \exp(\alpha_{\lambda_t} \bar{x}_t) \quad (17)$$

For each experiment, the joint and conditional tests will be computed.

3.4 Data Requirements and Sources

As emphasized, the main source of data required for the successful completion of this study is the Monte Carlo simulations method. The choice of Monte Carlo simulations method as the data-generating tool for this study is based on the fact that it can be adjusted to suit our analysis under the various assumptions proposed in this study. Prominent studies in panel econometrics among which include Holly and Gardiol (2000), Roy (2002), Phillips (2003) and Baltagi, Bresson and Pirotte (2006) and Baltagi, Jung

and Song (2008) also adopted this data generating process. However, after validating the proposed extensions, we can then apply the derived LM tests to selected panel surveys.

4.0 Outline of the Study

1.0 Chapter One: Introduction

2.0 Chapter Two: Literature Review

3.0 Chapter Three: Theoretical Framework and Methodology

4.0 Chapter Four: Results

5.0 Chapter Five: Discussion

6.0 Chapter Six: Conclusion

References

- Amemiya, T. (1971), The Estimation of the Variances in a Variance-Components Model. *International Economic Review* 12, 1-13.
- Avery, R.B. (1977), Error Components and Seemingly Unrelated Regressions, *Econometrica* 45, 199-209.
- Balestra, P. (1973), Best Quadratic Unbiased Estimators of the Variance-Covariance Matrix in Normal Regression. *Journal of Econometrics* 1, 17-28.
- Balestra, P. and M. Nerlove (1966), Pooling Cross Section and Time Series Data in the Estimation of a Dynamic Model: The Demand for Natural Gas, *Econometrica* 34, 585-612.
- Baltagi, B.H. (2008), *Econometric Analysis of Panel Data* (6thEdition, Wiley, Chichester).
- Baltagi B.H., B.C. Jung and S.H. Song (2008), Testing for Heteroscedasticity and Serial Correlation in a Random Effects Panel Data Model. *Working Paper* number 111, Syracuse University, USA
- Baltagi, B.H., G Bresson and A Pirotte (2006), Joint LM test for Heteroscedasticity in a One-Way Error Component Model. *Journal of Econometrics*, 134, 401-417.
- Baltagi, B.H., (2005), *Econometric Analysis of Panel Data*. (5thEdition Wiley, Chichester).
- Baltagi, B.H. and Q Li (1995), Testing AR(1) against MA(1) Disturbances in an Error Component Model. *Journal of Econometrics* 68, 133-151.
- Baltagi, B.H. and Q. Li, (1991), A Joint Test for Serial Correlation and Random Individual Effects. *Statistics and Probability Letters* 11, 277-280.
- Baltagi, B.H. and J.M. Griffin (1988), A Generalized Error Component Model With Heteroscedastic Disturbances, *International Economic Review* 29, 745-753.
- Baltagi, B.H., (1988), An Alternative Heteroscedastic Error Component Model, *Econometric Theory* 4, 349-350.
- Baltagi, B.H.(1980), On Seemingly Unrelated Regressions With Error Components, *Econometrica* 48,1547-1551.
- Bera, A.K., W. Sosa-Escudero and M. Yoon (2001), Tests for the Error Component Model in the presence of Local Misspecification, *Journal of Econometrics* 101, 1-23.

- Bhargava, A., L. Franzini, and W. Narendranathan (1982), Serial Correlation and the Fixed Effects Model, *Review of Economic Studies* 49, 533-549.
- Breusch, T.S. and L.G. Godfrey (1981), A Review of Recent Work on Testing for Autocorrelation in Dynamic Simultaneous Models, in: D.A. Currie, R. Nobay, and D. Peel, eds., *Macroeconomic Analysis: Essays in Macroeconomics and Economics* (Croom Helm, London).
- Breusch, T.S. and A.R. Pagan (1980), The Lagrange Multiplier Test and its Application to Model Specification in Econometrics. *Review of Economic Studies* 47, 239-254.
- Breusch, T.S. and A.R. Pagan (1979), A Simple Test For Heteroscedasticity and Random Coefficient Variation, *Econometrica* 47, 1287-1294.
- Breusch, T.S. (1980), Useful Invariance Results for Generalized Regression Models, *Journal of Econometrics* 13, 327-340.
- Burke, S.P., L.G. Godfrey, and A.R. Termayne (1990), Testing AR(L) Against MA(1) Disturbances In The Linear Regression Model: An Alternative Procedure, *Review of Economic Studies* 57, 135-145.
- Chamberlain, G. and Z. Griliches (1975), Unobservables with a Variance-Components Structure: Ability, Schooling and The Economic Success Of Brothers, *International Economic Review* 16, 422-450.
- Galbraith, J.W. and V. Zinde-Walsh (1995), Transforming the Error-Component Model for Estimation with General ARMA disturbances, *Journal of Econometrics* 66, 349-355.
- Harville, D.A. (1977), Maximum Likelihood Approaches to Variance Component Estimation and to Related Problems. *Journal of the American Statistical Association* 72, 320-338.
- Hemmerle, W.J. and H.O. Hartley (1973), Computing Maximum Likelihood Estimates for the Mixed A.O.V. Model using the W-Transformation. *Technometrics* 15, 819-831.
- Henderson, H.V. and S.R. Searle (1979), Vet and Vech Operators for Matrices, with some uses in Jacobians and Multivariate Statistics, *Canadian Journal of Statistics* 7, 65-81.
- Holly, A and L. Gardiol (2000), A Score Test for Individual Heteroscedasticity in a One-Way Error Components Model, Chapter 10 in J. Krishnakumar and E. Ronchetti, eds., *Panel Data Econometrics: Future Directions* (North-Holland, Amsterdam), 199-211.

- Hong, Y. and C.D. Kao (2004), Wavelet-Based Testing for Serial Correlation of Unknown Form in Panel Models, *Econometrica* 72, 1519–1563.
- Hsiao, C. (1986), *Analysis of Panel Data*. First Edition, Cambridge University Press. Cambridge).
- Hsiao, C. (1985), Benefits and Limitations of Panel Data. *Econometric Views* 4, 121-174.
- Hsiao, C. (2003), *Analysis of Panel Data*. Second Edition, Cambridge University Press. Cambridge).
- Kouassi E. and Kern O. Kymn (2008), Prediction in the Two-Way Random Effect Model with Heteroscedasticity. *Journal of Forecasting*, 27, 451-463.
- Kouassi E, Sango J, Bosson B JM, Teubissi FN and Kern O Kymn (2009) ‘Prediction from the Regression Model with the Two Way Error Components’, Forthcoming, *Journal of Forecasting*,
- Lejeune B. (2006), A Full Heteroscedastic One-Way Error Components Model for incomplete Panel: Maximum Likelihood Estimation and Lagrange Multiplier Testing, CORE Discussion Paper 9606, Universite Catholique de Louvain, 1–28.
- Klevmarcken, N.A. (1989), Panel Studies: What Can We Learn from Them? Introduction. *European Economic Review* 33, 523 – 529.
- Li, Q. and T. Stengos (1994), Adaptive Estimation in the Panel Data Error Component Model With Heteroscedasticity Of Unknown Form, *International Economic Review* 35, 981-1000.
- Lillard, L.A. and R.J. Willis (1978), Dynamic Aspects of Earning Mobility, *Econometrica* 46, 985–1012.
- Tim Lloyd, Oliver Morrisey and, Robert Osei (2001), Problems with Pooling in Panel Data Analysis for Developing Countries: The Case of Aid and Trade Relationships. Centre for Research in Economic Development (CREDIT), University of Nottingham Research Paper No. 01/14.
- Maddala, G.S., (1971), The Use of Variance Components Models In Pooling Cross-Section and Time Series Data, *Econometrica* 39, 341-358.
- Maddala, G.S. and T.D. Mount (1973), A Comparative Study Of Alternative Estimators For Variance Components Models Used In Econometric Applications, *Journal Of The American Statistical Association* 68, 324-328.

- Magnus, J.R. (1982), Multivariate Error Components Analysis Of Linear And Nonlinear Regression Models By Maximum Likelihood. *Journal of econometrics* 19, 239-285.
- Magnus, J.R. (1980), Multivariate Error Components Analysis of Linear and Nonlinear Regression Models by Maximum Likelihood, Discussion Paper No. 8&34 (University of British Columbia, Vancouver).
- Magnus, J.R. (1978), Maximum Likelihood Estimation of the GLS Model With Unknown Parameters In The Disturbance Covariance Matrix. *Journal of Econometrics* 7, 281– 312.
- Mazodier, P. and A. Trognon (1978), Heteroscedasticity and Stratification In Error Components Models, *Annales de l'INSEE* 30–31, 451–482.
- Mundlak, Y. (1978), On The Pooling Of Time Series And Cross Section Data, *Econometrica* 46, 69-86.
- Nerlove, M. (1971a), Further Evidence on the Estimation of Dynamic Economic Relations from A Time Series of Cross Sections, *Econometrica* 39, 359-382.
- Nerlove, M. (1971b), A Note On Error Components Models, *Econometrica* 39, 383-396.
- Nicholls, D.F., A.R. Pagan, and R.D. Terrell (1975), The Estimation and Use Of Models With Moving Average Disturbance Terms: A Survey, *International Economic Review* 16, 113-134.
- Phillips, R.F. (2003), Estimation of a Stratified Error-Components Model. *International Economic Review* 44 (2), 501–521.
- Randolph, W.C. (1988), A Transformation For Heteroscedastic Error Components Regression Models. *Economics Letters* 27, 349–354.
- Rao, S.R.S., Kaplan, J., Cochran, W.C. (1981), Estimators For The One-Way Random Effects Model With Unequal Error Variances. *Journal of the American Statistical Association* 76, 89–97.
- Rao, C.R. and S.K. Mitra (1971), *Generalized Inverse of Matrices and Its Applications* (Wiley, New York).
- Rilstone, P. (1991), Some Monte Carlo Evidence on The Relative Efficiency Of Parametric And Semi parametric Egls Estimators. *Journal of Business and Economic Statistics* 9, 179–187.
- Roy, N. (2002), Is Adaptive Estimation Useful For Panel Models With Heteroscedasticity In The Individual Specific Error Component? Some Monte Carlo Evidence. *Econometric Reviews* 21, 189–203.

- Swamy, P.A.V.B. and S.S. Arora (1972), The Exact Finite Sample Properties Of The Estimators of Coefficients In The Error Components Regression Models, *Econometrica* 40, 261-275.
- Verbon, H.A.A. (1980), Testing For Heteroscedasticity In A Model Of Seemingly Unrelated Regression Equations With Variance Components. *Economics letters* 5, 149–153.
- Wallace, T.D. and A. Hussain (1969), The Use Of Error Component Models In Combining Cross-Section With Time Series Data, *Econometrica* 37, 55-68.
- Wansbeek, T.J. (1989), An alternative Heteroscedastic Error Components Model, *Econometric Theory* 5, 326
- Wansbeek, T.J. and A. Kapteyn (1982), A Simple Way to Obtain the Spectral Decomposition of Variance Components Models for Balanced Data, *communications in statistics* vol. 11, 2105–2112.