Modelling size and illiquidity in West African equity markets

Bruce Hearn
Sir John Cass Business School, and
King's College London

and

Jenifer Piesse*
King's College London, and
University of Stellenbosch, RSA

Abstract

This paper assesses the effectiveness of traded turnover and Amihud (2002) constructs in measuring illiquidity which is used in constructing a multifactor CAPM. The performance of this model is contrasted against GARCH and simple stochastic drift models on a unique sample of five West African equity markets: BRVM, Ghana, Nigeria, Morocco and Tunisia together with London and Paris. Analysis of portfolio characteristics reveal that investment strategies centred on Francophone markets outperform those of Anglophone markets despite their lower mean returns. There is some evidence of limited benefits to investors from inclusion of the very small and highly illiquid BRVM and Ghanaian markets.

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* Corresponding author: Department of Management, King's College London, 150 Stamford St, London SE1 9HN. Tel: 44(0)207 848 4164. Email: jenifer.piesse@kcl.ac.uk

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This paper assesses the effectiveness of traded turnover and Amihud (2002) constructs in measuring illiquidity which is used in constructing a multifactor CAPM. The performance of this model is contrasted against GARCH and simple stochastic drift models on a unique sample of five West African equity markets: BRVM, Ghana, Nigeria, Morocco and Tunisia together with London and Paris. Analysis of portfolio characteristics reveal that investment strategies centred on Francophone markets outperform those of Anglophone markets despite their lower mean returns. There is some evidence of limited benefits to investors from inclusion of the very small and highly illiquid BRVM and Ghanaian markets.

1. INTRODUCTION

The rapid establishment and development of equity markets in developing economies since 1990 has been associated with the desire to benefit from foreign portfolio investment and provide an attractive venue for companies seeking to raise funds for much needed industrial and development projects. However issues relating to extreme illiquidity and high risk premiums are cited as major concerns of both potential investors as well as firms seeking to raise capital from cheaper sources and diversify ownership through a domestic listing (Lesmond, 2005; Hearn et al., 2009). West African securities markets are a particularly interesting sub-region of Sub Saharan Africa for study given that Morocco and Tunisia are two of the most developed markets in the entire continent, Nigeria has the highest number of listed firms in Africa, and the regional bourse in Cote d'Ivoire is unique in being a fully integrated market for member states of the Francophone West African Monetary Union (UMEAO[1]). There is also a clear division between West African states with a French civil code legal regime and those with institutions based on English common law (La Porta et al, 2008) which has caused regional financial integration initiatives to falter.

Liquidity by its very nature has proved a difficult concept to define. Much of this difficulty has arisen through its ability to transcend a number of transactional properties of markets including tightness, depth, resiliency (Lesmond, 2005) and information (O'Hara, 2003). Empirically defined constructs designed to capture this phenomenon centre on measurement of direct trading costs, such as tightness, which is measured by the bid-ask spread (quoted or effective) and indirect trading costs, linked to depth and resiliency, which are often represented by price impact measures. The lack of reliable and consistent bid-ask quotes in many emerging markets infers the use of market activity proxies in capturing liquidity. However there is little consensus regarding the applicability and efficacy of

[1] Union Monétaire et Économique de l'Afrique de l'Ouest (UMEAO) countries include Cote d'Ivoire, Benin, Togo, Burkina Faso, Mali, Niger, Senegal and Guinea-Bissau

ubiquitous measures such as turnover and the more recently developed price impact variable of Amihud (2002) (Lesmond, 2005). Consequently the first part of this study is devoted to testing these two measures against the quoted bid-ask spread, where available, to determine each measure's efficacy in estimating the underlying liquidity. Regression based tests provide evidence of the higher association between the Amihud (2002) measure and the total trading costs, defined as bid-ask spread plus trading fees for the three West African markets with available data, i.e. BRVM, Morocco and Tunisia as well as London and Paris.

While the literature concerning the importance of liquidity been prominent for over a decade research concerning liquidity risk and its applications is much more recent. Pastor and Stambaugh (2003) find evidence that investors employing leverage and facing solvency constraints do require higher expected returns for holding assets that are difficult to sell when aggregate liquidity is low. Furthermore stocks with a higher sensitivity to aggregate liquidity generate higher returns than low-sensitivity stocks inferring that liquidity is an important state variable for asset pricing. The inability of the traditional CAPM and the three factor augmented CAPM of Fama and French (1993), seeking to describe the cross section of asset returns with additional size and book-to-market factors, in capturing liquidity effects represents a serious caveat in asset pricing (Liu, 2006). Liu (2006) in line with Daniel and Titman (1997) finds considerable evidence of the limited explanatory power of the Fama and French model in capturing the cross section of asset returns. Martinez et al (2005) also present evidence of the limited explanatory power of the Fama and French three factor model although there is some evidence of some explanatory power in retaining the size factor. In addition to questions regarding the benefits of including the book-to-market variable there are serious limitations in obtaining consistent accounting book values of firms from emerging markets. Furthermore emerging stock markets are highly skewed with many being dominated by a handful of large firms with the rest of market being populated with SMEs. As such a size factor should be retained within the pricing model in explaining the cross section of returns. This study finds evidence that the liquidity and size factors are significant in explaining cross section of returns and outperforms the traditional CAPM. In line with the findings of Liu (2006), the state nature of liquidity as well as the conjecture that liquidity risk better captures firm distress risk, and the associated solvency issues, than the combination of size and book-to-market factors used in Fama and French model, explains the improved performance of the size and liquidity augmented model used in this work. The success of this multifactor CAPM provides support for the continued use of the risk-return paradigm in asset valuation.

Three approaches are used to model aggregate market-wide total returns indices: a simple stochastic drift, or iid, model, which implies a stochastic time series with drift; the generalised autoregressive conditional heteroskedasticity (GARCH) model developed by

Bollerslev (1987); and the size-liquidity augmented CAPM developed in this paper. When the models have been specified, the one step-ahead forecast of mean and variance-covariance is be used within an optimised portfolio framework to assess the potential costs and benefits of diversification for investors.

There is a considerable literature that uses GARCH and related market models of GARCH-in-Mean (GARCH-M) models to capture time series effects within equity price series. Much of this owes its foundations to the work of Bollerslev (1987). It then was extended to include GARCH models to capture the dynamics of autoregressive means and volatility processes for valuation in stock options (Ritchken and Trevor, 1999) as well as a variety of applications in the areas of equity indices and stock price volatility (Engle, 2001). In addition, this literature itself has spawned the more recent related families of ARCH/ GARCH type models such as Exponential GARCH or EGARCH (Nelson, 1991). However, this paper models the conditional means of series through an autoregressive form augmented with simple generalised conditional heteroskedasticity techniques.

Modern portfolio theory indicates that investors should hold mean-variance efficient portfolios of assets. However, one recent strand contrasts the ability to forecast means and covariances in terms of the consequent effects on optimal portfolio asset weights. Harvey (1994) compared the predicted mean and covariance matrix from a simple stochastic drift model, inferring an unconditional asset allocation strategy where managers have no other information other than historic returns to predict future values to regression models that include a variety of world and local market variables as conditioning information. The analysis focuses on exploring the benefits to investors of diversification into emerging markets through the development of efficient frontiers. Chan et al (1999) assess the forecasting ability of a variety of models both in terms of variance tracking performance to a known benchmark as well as in relative portfolio weights in a minimum variance optimised portfolio setting. The focus is directed towards individual industries within developed OECD markets. In a further paper, Harvey (1995) follows a similar route but concentrates on market index level analysis. This study focuses in contrasting the performance of portfolios including markets with legal and regulatory regimes based on French civil or English common law which are predominant in West African region. In line with La Porta et al (2008) concerning the effects of different legal regimes on the degree to which firms raise external finance there are considerable differences in the mean-variance performance of portfolios.

This paper is structured as follows. Section 2 has three distinct parts: the first provides an overview of the institutional features of West Africa's markets, the second introduces the liquidity measures and their construction, while the third discusses data specific issues. Section 3 outlines the three approaches to modelling the total returns series, the simple stochastic drift, or iid, GARCH, and the three-factor size and liquidity augmented

CAPM, as well as describing the application of mean-variance optimisation techniques. Section 4 discusses the empirical results. The final section concludes and provides development policy inferences from the evidence presented in this paper.

2. WEST AFRICAN EQUITY MARKETS AND LIQUIDITY MEASUREMENT

(i). West Africa's securities markets

The principal characteristics of these markets are summarised below (see Piesse and Hearn (2005) for an extended discussion of African stock markets):

BRVM (Bourse Regionale des Valeurs Mobilieres)

The Bourse Regionale des Valeurs Mobilieres, or BRVM, was originally established in 1973 as the Bourse de Abidjan, in Cote d'Ivoire. Given the existing economic and monetary union across Francophone West African countries, UMEAO, the focus of the local exchange was extended in 1998 to a regional basis and participation was encouraged, both in new listings as well as investment, from across the UMEAO. The trading system is electronic with remote terminals installed in each of the licensed brokers, or Societe de Gestion et d'Intermédiation (SGI). Trading is undertaken five days per week and starts with a pre-opening call auction followed by continuous auction from 10-00 to 10-30am. Nine brokers, or SGIs, are based locally in Abidjan, with another three in Dakar, Senegal where the regions central bank is located, and another four in Benin. All other UMEAO countries have one SGI, which as in the case of Mali has been established jointly between the local finance communities. Technical issues relating to exchange activities and marketing of the exchange are handled by a separate Antennae de Bourse, which is located in each UMEAO country. Settlement is in partially G30[2] compliant and there are a small number of well capitalized international custodian banks in the market, including affiliates of Societe Generale, BNP Paribas, Credit Agricole and Citigroup. Trading activity and market capitalization, as shown in Table 1, is highly concentrated with Sonatel, the Senegalese telecommunications company, accounting for 53.95% of traded value and 46.51% of capitalization. Although the microstructure of the regional bourse has been designed to precipitate the maximum order flow possible in one of the poorest regions of the world the evidence from Table 2 suggests that it offers institutional

[2] G30 relates to the Group of Thirty which is the most influential body to encourage the standardisation and improvement in global securities administration. Following a symposium in London in March 1989, the following recommendations were agreed: i) Brokers should match trades on day after deal date (T+1); ii) Trade confirmation on trade day plus 2 days (T+2); iii) Central Depository for safe keeping of shares; iv) Net basis settlement of cash and stock; v) Settlement takes place as delivery vs. payment or receipt vs. payment; vi) Settlement in same day funds; vii Settlement effected on trade date plus 3 days (T+3) 8; viii) Securities lending should be permitted; ix) International securities numbering system must be adopted (ISIN code).

investors in the UMEAO minimal opportunities for diversification. Table 2 indicates that average annual order flow from Mali accounts for under 2% of traded value on the regional bourse which itself is one of the least liquid in Africa. Furthermore this figure is overwhelmingly dominated by a single individual investor who accounts for over 90% of trading activity amongst the tiny retail investment community. This adds further support to the assertion in Lavelle (2001) that the BRVM acts to further concentrate wealth in the hands of local elites rather than as a redistribution mechanism enforcing high governance standards through diversified ownership.

Tables 1 and 2

Ghana

The Ghana stock exchange was established in 1989 as part of the transition from socialist central planning under the guidance of the international financial institutions. Trading that was originally undertaken three days per week has now become continuous from Monday to Friday between 10-00 and 11-00am. This is centred on a trading floor and uses a simple form of continuous auction system where bid and ask orders are written manually on a series of boards. However although the market is partially G30 compliant and information is disseminated through Bloomberg and Reuters, regulation is weak with trades and prices often being agreed informally and the market institutions merely being used to announce pre-agreed details (Akotey, 2008). The market is highly concentrated with five stocks accounting for 79.08% of traded value and 67.15% of capitalization, as demonstrated in Table 1. Small local financial institutions seeking to diversify their balance sheets in accordance with international best practice are responsible for 64.84% of listed equity. Settlement is undertaken centrally using exchange facilities and there are plans for a central securities depository, CSD.

Nigeria

The Nigerian stock exchange was originally established in 1960 as the Lagos stock exchange with 19 listed firms. The exchanges remit was extended as part of the governments privatization programme with additional branches opening in major commercial cities around the country including Kaduna (1961) and Abuja (1999) (Nigerian Stock Exchange, 2009). Trading is undertaken using an electronic automated trading system (ATS) on a daily basis from 11-00am to 14-00pm and settlement is partially compliant with G30 requirements due to the presence of a CSD, created in 1992, and international custodian banks. The network of 219 brokers alongside the 234 listed firms ensures it is the largest market in West Africa and although trading activity and capitalization is less concentrated than its neighbouring markets the financial sector accounts for 59.74% of market capitalization, as seen in Table 1.

Information dissemination is undertaken through stock exchange website and Reuters as well as local business journals.

Morocco

The Moroccan stock market, the Bourse de Casablanca, was established in 1929, making this market one of the oldest in North Africa. The exchange has progressed through several phases of development and in 1997 adopted an electronic trading system based on order matching located centrally. The trading system was improved further in 2001 to facilitate delocalised trading from the offices of the local brokers. MAROCLEAR, the national central securities depository, was established in 1998 for settlement, securities transfer and payment and to minimise operational risks. This became fully G30 compliant by 2001 with settlement versus delivery occurring on trade date plus three working days (Bourse de Casablanca website, 2008). Trading is reported electronically to market participants and to international data vendors such as Bloomberg and Reuters. This gives the market the opportunity to attract overseas investors. Stock market awareness is high and the exchange is used as a successful route for domestic flotation, although it also attracts significant retail and institutional investors.

Tunisia

The Bourse de Tunis was established in 1969 and demutualized in 1995 with controlling shareholders being the stakeholders in the market, i.e. the local brokers, of which there are 24. Trading was migrated to an electronic system, NSC, under the guidance of Euronext with outlets installed in all licensed members in 1996, and is further split into fixing and continuous systems, with the former handling small and illiquid securities and being comprised of a series of sequential electronic call auctions (Bourse de Tunis website, 2009). Trading hours in continuous market are 9-00am to 14-10pm in the months outside July, August and Ramadan where hours are 8-30am to 12-10pm. Settlement is fully compliant with G30 guidelines. However there is little domestic stock market culture and only an estimated 5% of finance raised by firms in 2007 was done so through the stock exchange (Zribi, 2008).

(ii). Liquidity constructs

The Bid Ask spread and commission cost

The data on the end of month bid and ask quotes were collected from Datastream for London, Paris and Morocco, Bloomberg for Tunisia, and the stock exchange website for BRVM. Data was unavailable for Ghana or Nigeria. There is considerable variation in the length of

intraday data available with that for London, Paris, Morocco and Tunisia all being available for over 15 years. Intraday data for the BRVM is available since inception in 1998 from the stock exchange website. Owing to inconsistencies between the various data sources it is also necessary to corroborate data with that obtained direct from the individual markets themselves. The bid-ask spread is calculated using the average of the available monthly quotes and incorporates at a minimum a single month's quote for that month. The average bid-ask spread spanning the quarter is used for the estimate of the spread. This procedure minimizes outlier problems and averages out the recording of either highs or lows in quotes resulting from monthly sampling. Finally in line with Lesmond (2005) bid-ask spreads that exceed 80% are trimmed as these are deemed to have resulted from coding errors. The monthly quoted spread is defined as:

$$Quoted\ spread_M = 1/2\left[\left(\frac{(Ask_M - Bid_M)}{(Ask_M + Bid_M)/2}\right) + \left(\frac{(Ask_{M-1} - Bid_{M-1})}{(Ask_{M-1} + Bid_{M-1})/2}\right)\right] \quad (1)$$

In order to estimate the total trading transaction costs, the costs associated with a round trade, or opening and subsequently closing a position are added on to the quoted spread for each month. Brokerage and Exchange fees are calculated from the fee schedules detailed in Appendix 1. When a percentage commission fee is not provided the maximum fixed cost is applied to the aggregate daily traded value data.

Turnover

Daily trading volume data and shares outstanding data was obtained from a combination of Bloomberg, Datastream, and the national stock exchanges. At first glance it is apparent that there is considerable variation in this measure on an intra-market basis. This reflects the substantial differences in both liquidity and turnover for many of the companies within each market. Any turnover statistics that exceed 100% of the shares outstanding in any month are trimmed from the sample. The shares-outstanding is determined at the start of the year and remains constant for the 12 months thereafter. The daily turnover measure is defined as:

$$1\Big/ D_M \sum_{t=1}^{M} (volume_t / \text{shares - outstanding}) \quad (2)$$

where D_M is the number of days in the month, M.

Amihud (2002) measure

Daily price and volume data are sourced from Bloomberg, Datastream and the national stock exchanges. The daily security prices are scanned for data errors, omissions and delistings. Following the procedure outlined in Lesmond (2005) the prices are used calculate daily returns. To control for return outliers, a data error filter eliminates daily prices that are +/- 50% of the prior day's price and that day's price as well as previous day's price are deleted

from sample. Equally if zero volume occurs on day t, then that day is deleted from average. Finally the measure is multiplied by 10^6 as undertaken in Amihud (2002) in order to provide a common representation of measures and facilitate comparison. The Amihud measure is defined as:

$$1\Big/ D_M \sum_{t=1}^{M} \left(| R_t | / \Pr ice_t \times Volume_t \right) \qquad (3)$$

(iii). Data: Sources

Daily stock closing, bid and ask prices, total number of shares outstanding, traded volumes, dividend per share in local currency and converted into UK£ were obtained for London, Paris and Morocco from Datastream. These variables were sourced from both Bloomberg and the national stock exchanges for Tunisia, Nigeria, Ghana and BRVM. These data formed the basis of calculation of the daily return variance, or volatility, market capitalization, defined as total number of shares outstanding multiplied by daily closing price, and various liquidity constructs. The total returns series for each stock were sourced direct from Datastream for London, Paris and Morocco while they had to be constructed using the procedures employed by Standard & Poors in assuming reinvestment of dividends and taking account of stock splits, rights issues and other corporate actions affecting a stocks intrinsic value. Exchange rate and UK- Gilt/Treasury yield data are sourced from Datastream. The one-month UK-Gilt/Treasury Bill yield rate represents the risk free rate although this is adjusted to take account of monthly excess returns as opposed to the quoted equivalent annualised rates. The conversion of the total returns series and prices into sterling and the use of UK - Gilt/Treasury yield rate assumes long term parity between individual domestic currencies and sterling. In many cases companies were deleted from sample owing to either data inconsistencies or the lack of availability of certain variables that rendered the generation of total returns impossible. Nigeria is one example where there are 234 listings yet 60 of these do not have data and a further 45 firms are missing one critical determinant needed for the generation of total returns indices. Consequently the sample size for Nigeria is 129 firms.

(iv). Data: Summary statistics relating to liquidity measures

The skewed nature of West Africa's equity markets in terms of trading activity and capitalization is highlighted in Table 3. This contrasts the mean cross section values for daily percentage zero returns, stock prices, traded volumes, market capitalization and bid-ask spreads for the component firms within London FTSE100 and Paris CAC40 indices as well as the West African markets. The latter are further sub-divided with the markets of BRVM, Ghana and Nigeria being split into three groups: a total number of firms for which consistent data is available, and then the top 10 and top 5 firms as ranked by market capitalization.

Morocco and Tunisia are further sub-divided into two groups: an overall group, dependent on where data is consistently available, and a top 10 grouping based on market capitalization. There is clear evidence of a size effect in all markets, with the mean cross sectional capitalizations of the top 5 and top 10 groupings of firms being several orders of magnitude larger than the overall market. Similarly the bid-ask spread, representing liquidity, decreases as mean cross sectional firm size increases indicating the association between size and liquidity, although this is not the case for BRVM where bid-ask spread increases as firm size increases from value of 0.0592 for cross section of 33 firms to 0.101 for top 5 firms. The percentage of zero daily returns variable, another measure of liquidity, in line with the general trend of bid-ask spreads, also decreases from the aggregate market groupings of firms to the top 10 and top 5 groups of firms. However the greatest degree of illiquidity that is fairly consistent across all groupings of firms is in BRVM and Ghana. Ghana has a percentage daily zero returns of over 77% for the overall market that decreases to a mere 72.24% for the top 5 firms, while BRVM is the most illiquid market with 89.65% for the overall market and 80.23% for the top 5 firms demonstrating the severe price rigidity present in these markets.

Table 3

An assessment of the correlations between the key variables affecting the bid-ask spread and liquidity is provided in table 4. Given the variables have different measurement scales a non-probability distribution limited correlation, the Spearman's rank, is used for the measurement of association. In addition to the bid-ask spread and the two liquidity measurement constructs, turnover and Amihud variables, price, daily price return volatility, traded volume, and market capitalization measures are introduced as control variables in line with those used in the investigations for price of trading immediacy undertaken by Stoll (2000). The rationale for the inclusion of these variables is based on order processing and inventory considerations of traders, albeit in the US equity market. Increases in volume and firm size increase the probability of locating a counterparty, thereby mitigating the risk of accepting and holding inventory risk. The stocks price volatility, in local currency terms, provides an indication of the risk of adverse price changes of a stock placed on a trader's inventory, while the price measure itself controls for the effect of discreteness and is an additional proxy for risk in that low price stocks tend to be riskier. In this light traded volume is significantly, as indicated by correlations of over 50%, positively correlated to market capitalization in all markets except BRVM. The BRVM is also unique in having a positive correlation between stock price and market capitalization which infers that large stocks have higher prices as expected. In terms of the liquidity measures and the Amihud construct is highly positively correlated to the bid-ask spread in London and Paris while significantly negatively correlated to both market capitalization and traded volume for London and Paris as well as Tunisia. This is intuitively

expected as any trading activity in stocks which are generally less traded and have lower market capitalizations consequently has a higher impact on price, which is measured by the Amihud construct. Counter intuitively in the case of BRVM and Morocco traded volume has a positive correlation with the Amihud measure indicating that more highly traded stocks have greater impacts on price, although this may be an effect of severe illiquidity in these markets. The turnover variable is positively correlated to traded volume in the markets of Morocco, Tunisia and BRVM indicating as expected that increased trading volume leads to higher values of turnover, which is in line with the volume-based focus of this liquidity measure.

Table 4

3. EMPIRICAL MODELS

This section considers first a simple naive unconditional modelling strategy, that of iid, followed by two conditional modelling strategies, namely GARCH and the three-factor CAPM.

(i). IID model: Unconditional asset allocation

This strategy is considered unconditional as it implies that there is no other information relevant for forecasting the next period's price other than the previous price, that is, stock returns are not predictable. The expected returns are modelled as a rolling window of the mean returns over the previous 12 months. Despite the movement in these mean returns as the 12-month window moves, using the average returns assumes that the best forecast of the equity returns is its past average. This is consistent with the random walk model of stock prices with a drift component and has a core assumption that the underlying time series is weak form efficient.

The 12 month rolling window is extended for the calculation of standard deviation, and hence variance, as well as correlations from which updated covariance estimates are calculated. The simple model can be represented by

$$y_t = \mu + \varepsilon_t \ldots\ldots \varepsilon_t \sim N(0, \Sigma), \quad (4)$$

in which μ and Σ are matrices of constant parameters. Recursive rolling window estimation is necessary for μ and Σ to generate vectors of sample means and covariances.

(ii). GARCH model: Conditional asset allocation

As discussed in Bollerslev (1987), there is evidence that the change in prices and rates of return are approximately uncorrelated over time, but characterised by tranquil and volatile

periods. Allowing for such dependence this study takes the conditional mean as being dependent only on its first order lagged and a constant,

$$y_t = \alpha_0 + y_{t-1} + \varepsilon_t \tag{5}$$

together with a GARCH (p, q) model for the conditional variance:

$$E\left(\varepsilon_t^2 \mid \psi_{t-1}\right) = h_{t|t-1} \tag{6}$$

$$h_t = \omega_0 + \sum_{i=1}^{p} \delta_i h_{t-i|t-1-i} + \sum_{j=1}^{q} \gamma_j \varepsilon_{t-j}^2 \tag{7}$$

where $\omega > 0$, $\delta \geq 0$, $\gamma \geq 0$. It can be seen in (5) and (6) that there is a tendency for large (small) residuals to be followed by other large (small) residuals but of unpredictable sign. Equation (7) says that the value of h_t depends on past values of shocks (the q moving average terms), which are captured by the lagged squared residual terms, and on past values of itself (the p autoregressive terms), which themselves are captured by lagged h_t terms. Furthermore Bollerslev (1987) allows assumptions concerning mildly leptokurtic conditionally normal errors to be relaxed, and an adherence to conditionally t-distributed errors. This gives the GARCH model greater flexibility with financial time series with very fat-tailed distributions, which are typical in emerging market time series.

(iii). Size and Liquidity Augmented CAPM

In the spirit of the three-factor CAPM model of Fama and French (1993) this work follows the reasoning of the more recent work of Martinez et al (2005) and Shum and Tang (2005) in modifying the augmented factors to take account of size and liquidity effects that offer improved performance in capturing anomalies across the cross section of stock returns which are particularly prevalent in emerging markets. Thus, in addition to the market excess returns, the model is augmented by the excess returns attributed to size (SMB), and the excess returns attributed to illiquidity (ILLIQ).

The market, size and liquidity factors used in the CAPM model are formed through the universe of available stocks being sorted into equally weighted portfolios with rebalancing being undertaken each December of every year from 2002 to 2008 inclusive. All stocks are assumed to be held continuously for a further year following rebalancing. The market portfolio itself is the simple arithmetic mean of the cross section of total returns in the universe. This universe is sorted each December first by each stocks market capitalization with the subsequent formation of three size ranked portfolios, “Small”, “Medium”, and “Big”, and then each of the individual three size ranked portfolios is further sorted into another three portfolios based upon the liquidity measure. The size factor is formed from the cross sectional mean returns of the small size portfolio minus the big size portfolio and is referred to as the SMB factor, in line with the original notation of Fama and French (1993). Given the

Amihud liquidity construct outperforms the turnover construct in capturing the effects of liquidity within the sample group markets this is used in the ranking process of sorting stocks into portfolios based on their relative liquidity. Consequently the liquidity factor is formed from the mean of each of the three "High" illiquidity sorted portfolios, formed from having sorted the size portfolios, minus the mean of the "Low" illiquidity portfolios, resulting in a HML factor, in line with notation of Liu (2006). The construction of the market variable is complicated by the lack of appropriate regional benchmarks in the Sub Saharan African region overall and as a consequence is chosen to be comprised of the universe of all sample group markets.

One the three factors have been formed the three-factor CAPM can be restated as the expected return on a risky portfolio *p*, in excess of the risk free rate $E(R_p) - R_f$ is a function of (i) excess return on the market portfolio, $R_m - R_f$; (ii) the difference between the return on a portfolio of small-size stocks and of large-size stocks, SMB; and (iii) the difference between the return on a portfolio of high illiquidity stocks and of low illiquidity stocks, ILLIQ. Therefore, the expected excess returns on a portfolio *p* of emerging market stocks can be written as

$$E\left(r_{pt}\right) - r_{ft} = \beta_p \left[E\left(r_{mt}\right) - r_{ft}\right] + s_i E\left(SMB\right) + h_i \left(HML\right) \qquad (8)$$

The equilibrium relation of the Fama and French (1993) three factor model is stated in terms of expected returns. In order to test the model with historical data, it is necessary to transform (8) to the following estimating equation:

$$r_{it} - r_{ft} = \alpha_i + \beta_i (r_{mt} - r_{ft}) + s_i SMB_t + h_i HML_t + \varepsilon_{it} \qquad (9)$$

where the variables are described above and $\varepsilon_{p,t}$ is an iid disturbance term.

In all cases, unconditional and conditional, mean-variance optimised minimum variance portfolios were formed using the techniques detailed in Appendix 2.

4. RESULTS

(i) An assessment of liquidity measures ability in explaining total costs

A more direct measure of association between the liquidity constructs and control variables, as defined in Stoll (2000), is provided through cross sectional panel regressions in Table 5. Interestingly the control variables, namely price, volatility, traded volume, and capitalization or size, are consistently statistically significant in the London and Paris markets. Thereafter there is significance of the traded volume variable in each regression for Morocco and only occasional significance of this in the case of Tunisia. Although the turnover liquidity construct was statistically significant in explaining total trading costs for London, Paris and Tunisia it had little impact on the overall explanatory power of these models in contrast to the

inclusion of the Amihud construct. The coefficients for the Amihud variable also have higher values and a larger and more obvious association. However while explanatory power was highest in the two European markets of London and Paris, in excess of 60%, Morocco and BRVM models had lower explanatory power of between 30% and 35% with Tunisian models having the lowest explanatory power of under 10% indicating that none of the variables have a major impact on Tunisian total trading costs or liquidity.

Table 5

(ii). Summary statistics relating to size-liquidity sorted portfolios

Descriptive statistics for all nine size-illiquidity factor sorted portfolios and the zero-cost SMB and ILLIQ portfolios are in Table 6. The average mean returns increase considerably from large to small size stock portfolios. This is also reflected in the measure of volatility, where standard deviations increase dramatically from larger size firm to smaller size firm portfolios. Average returns in small size stock portfolios tend to be more risky than in larger stock portfolios, but also have higher potential returns. However the negative value of the mean of the SMB indicates the likelihood of a reverse size effect from that in Fama and French (1993) where returns steadily decrease as stock size increases. Although there is little difference between the low and high liquidity portfolio means, there is an increase in volatility from low illiquidity to high illiquidity stock portfolios. Even in a less liquid market this result is expected since the impact of sudden erratic order flow on stock prices reflects significant adjustments in value where there is occasional trading activity.

London and Paris stocks dominate the large and medium size sorted portfolios while Nigeria dominates the medium and small size portfolios. Furthermore London stocks tend to be concentrated in the low illiquidity sorted portfolios while Parisian stocks congregate in medium and high illiquidity portfolios. Nigerian small size stocks tend to be low illiquidity while those of medium size are high illiquidity. Contrastingly Moroccan and Tunisian stocks tend to be small size, with only a few being medium size, and concentrated in high illiquidity portfolios.

Table 6

Generally these results are intuitively expected given London is a highly liquid international market and the stocks are the constituents of the prestigious FTSE100 index in terms of size and liquidity. Parisian stocks, themselves belonging to the CAC40 index, are also large but from a less liquid market than London. Nigeria has a wide range of capitalizations but in contrast to the European markets is smaller and much less liquid, while the firms in the Francophone markets of Morocco and Tunisia tend to raise less capital through stock markets, and hence have lower capitalizations and sizes.

(iii) Performance of traditional CAPM against three-factor CAPM

Table 7 reports the results from the grouped pooled regression on all nine size-illiquidity sorted portfolios. The Jensen alpha, α_p, from the one-factor CAPM is significantly different from zero in all cases with the exception of the small size-low illiquidity and medium size-medium illiquidity portfolios. In contrast the Jensen alpha from the three-factor CAPM is not significantly different from zero in every case except for three portfolios: medium size-medium illiquidity, medium size-high illiquidity, and big size-high illiquidity. This is indicative of an overall improvement in performance of the three-factor model in contrast to the simple CAPM as the non-significant alpha terms infer little segmentation between the portfolios and the aggregate market portfolio. The estimated coefficients on both the market excess return (β_p) and the illiquidity factor (H_p) are large and significant in almost all cases. Those on the size factor-mimicking portfolio (S_p) are smaller in the majority of cases and are only significantly different from zero in the large or small-size company portfolios. The coefficients on the large-size portfolios are negative as well as being highly statistically significant. The negative sign on the large-size portfolio betas indicates that large firms' returns decrease when the size premium increases, which is the opposite for small firms. This behaviour is not expected and is indicative of a reversal of the documented "size effect" that effects the valuation of smaller firms (Martinez et al, 2005). It is also a feature of an extremely heterogeneous universe of stocks, where there are considerable differences between the firms that comprise the FTSE100 and Paris CAC40 indices and those listed on the much smaller and highly illiquid markets of Morocco, Tunisia and Nigeria. This is the opposite of what would be expected and does not provide investors with good hedging opportunities. Thus, as with the results for the small-size portfolios, a different valuation method would be needed to price very high illiquidity stocks and firms accurately. The estimated coefficients on the illiquidity factor-mimicking portfolios are negative for low and medium-illiquidity portfolios indicating as expected that more liquid firms experience a decrease in expected returns when aggregate market illiquidity increases. In general, the coefficients on the low-illiquidity and medium-illiquidity portfolios are negative, as one would expect, with firms paying lower returns when the illiquidity variable increases. However, the coefficients on the high-illiquidity portfolios are positive indicating that these companies pay higher returns when the illiquidity measure increases. The increased explanatory power of these models illustrates that the augmented CAPM is appropriate for highly illiquid markets.

This is a very important result in the context of emerging markets, as the vast majority of research on the original of Sharpe (1964) and Lintner (1965) is confined to developed markets. In the Table, the first adjusted R^2 [Adj R^2 (1)] is the result from regressing the expected return on risky portfolio *p*, in excess of the risk free rate $E(R_p) - R_f$ as

a function of the excess return on the market portfolio, $R_m - R_f$. The second adjusted R^2 [Adj R^2 (3)] is the result from regressing the size and illiquidity augmented three-factor model on excess returns. In all size and illiquidity groups there is substantial improvement, in many cases by more than 100%. This provides further evidence that in a broad, market-wide context that considers stocks from all countries in this sample, the model has a good fit and the size and illiquidity factors are significant across the entire group.

Table 7

(iv) Modelling country portfolios and cost of equity estimation

Table 9 reports estimates of the cost of equity calculated from the expected returns from each country regression. It should be noted that the market portfolio used is restricted to a sample of largely small and illiquid African markets, composed of very small and volatile firms and consequently London and Paris are the only markets that are truly liquid and comprised of large firms with a low cost of equity. In contrast, the high cost of equity for the African markets is used as the discount factor and applied to future cash flows in project valuation. The cost of equity is calculated from the annualised combination of the total risk premium, which is the sum of market, size and illiquidity premiums, with the 1 month UK Treasury rate a proxy for the risk free rate.

Tables 8 and 9

Average Returns in the London Market

The London market is represented by the FTSE100 index. Companies follow the code of corporate governance stipulated by the Cadbury Report and Sarbanes-Oxley, which requires independent audit and timely reporting. Because of the high fixed costs involved in listing and compliance, only the largest, best performing and most heavily capitalised companies are included but benefit from low costs of equity. This is reflected in the absolute size of the coefficients in the UK portfolio in Table 8. In contrast to the other emerging markets, all coefficients are highly significantly different from zero, while the Jensen alpha, α_p, is not significantly different from zero in accordance with expectations from the model. The adjusted R^2 indicates that the model containing premiums from all three factors, market, size and illiquidity, explains over 90% of the variance. However the presence of a significant Jensen alpha, α_p indicates some degree of segmentation between the London market and Africa, which is expected. Given the London market is a global leader, the cost of equity is the lowest of this sample at 4.04%. Given the selection of countries included in the market portfolio the London market is expected to have an extremely low cost of equity value. Clearly, the London listed firms are distinct from the rest of the sample.

Average Returns in the Paris Market

The Parisian market is represented by the prestigious CAC40 index. Though liquidity and company size is smaller than that in London market participants adhere to rigorous OECD corporate governance directives as well as timely informational accounting and auditing disclosures. Fixed costs are high for firms seeking to comply with high level of regulation and governance standards inferred from listing, though are not as high as those associated with London. Bank-based finance also plays a more important role in firms seeking external finance for development (La Porta et al, 2008). Equally regulatory and legal systems are centred on French civil code law which is fundamentally different from that of Anglophone countries. Given these differences the coefficients on all factors, shown in Table 8, are not as strongly statistically significantly different from zero as those of London and the Jensen alpha, α_p, is only narrowly not significantly different from zero as implied by theory. The cost of equity is higher too with a discount value of 9.19%.

Average Returns in the Tunisian Market

The Tunisian market has a lower cost of equity (12.53%) than neighbouring Morocco which reflects the well designed market institutional environment established under the guidance of Euronext Paris. Regulatory and corporate governance standards mirror OECD standards and those prevailing in France. However the model has the lowest explanatory power with an adjusted R^2 of 0.2907 though coefficients are significantly different from zero apart from the Jensen alpha term. Although the Tunisian market is small alongside Morocco it has the most advanced level of market institutional development and corporate governance in North Africa and it is interesting to note that North African markets (Algeria, Tunisia and Morocco), all have French civil commercial legal codes and the regulatory systems are based on the French model. In addition, securities markets tend to be less developed compared with the banking system. Thus, estimation of the cost of equity and cost of capital for these countries may fail to include many variables that are relevant to the structure of their financial systems.

Average Returns in the Moroccan Market

The overall fit of the model is good for Morocco with all coefficients being statistically significant and the Jensen alpha, α_p, term not significantly different from zero. An adjusted R^2 of 0.4418 indicates that this model has moderate explanatory power and the model gives rise to an estimated cost of equity of 18.62% as shown (Table 9).

Average Returns in the Nigerian Market

The model has a high explanatory power for Nigeria, with an adjusted R^2 of 0.8461 and a Jensen alpha coefficient not significantly different from zero. The cost of equity for Nigeria

is the highest at 45.45%. The very high cost of equity is largely due to two factors. The first is the genuine paucity of regulation and corporate governance in the Nigerian market with many firms producing unaudited financial statements or producing those undertaken in accordance to Nigerian accounting standards as opposed to internationally recognised levels. This is a pervasive issue in a marketplace dominated by smaller firms that have difficulty in affording the high fixed costs in employing independent auditors to perform regular and timely accounting reporting. The second factor stems from the market being dominated by smaller, less well known and more illiquid firms whose stocks have high volatility which upwardly inflates market beta estimates in particular thereby giving rise to very high costs of equity. A more detailed discussion relating to this issue can be found in Hearn et al (2009).

(v) Performance of unconditional asset allocation strategy

The unconditional minimum variance portfolio performance results are reported in Table 10 and the recursively optimised portfolio holding weights are in Table 11. There are four potential portfolio combinations considered. The first two are the Anglophone and Francophone portfolios with the former being composed of London, Ghana and Nigeria and the latter of Paris, Morocco, Tunisia and BRVM. These represent the split of markets between the two predominant legal regimes: those of French civil code and English common law. The final two portfolio combinations contrast the difference in performance between an overall portfolio containing all markets with and then without the two highly illiquid markets of Ghana and BRVM. Analysis of investment Sharpe ratios that trade off a portfolios expected return against it's level of risk show that the Francophone portfolio, despite having a lower annual mean return that the Anglophone portfolio (18.38% for the former as opposed to 21.39% for the latter) has a higher Sharpe ratio (0.5366) in contrast to the English common law centred portfolio (0.4813). Interestingly the asset weights, shown in Table 11, are distributed between London and Ghana with Nigeria being minimal for the Anglophone portfolio. The Francophone portfolio tends to hold Parisian, Tunisian and Moroccan assets in approximately equal proportions while commonly maintaining a minimal holding of BRVM of less than 20%. However the portfolio with the highest Sharpe ratio (0.6792) is that of the overall portfolio including the two highly illiquid markets of Ghana and BRVM. This trades off a higher annual mean return (18.38%) with a lower risk (6.14%) against less preferable values (15.62% return and 6.65% risk, or standard deviation) for the overall portfolio not including Ghana and BRVM. Similarly analysis of asset weights in Table 11 shows that in both cases the portfolios are dominated by holdings of London, Morocco and Tunisia. Paris alongside Nigeria is held in minimal proportions. Ghana is held at a level of between 10% and 14% and BRVM is held under 10%. This infers that although Ghana and BRVM assets

are held minimally owing to their mean-variance characteristics their inclusion does have beneficial effects on the overall risk-return characteristics of the overall portfolio.

Tables 10 and 11

(vi). Performance of conditional asset allocation strategy

The parameters in the GARCH only models in Table 12 that have a simple first order autoregressive term and constant in the mean equation also present problems of acceptable confidence levels. In all cases the constant term is statistically significant at the 90% confidence level. However only in the cases of Nigeria (2, 1), Ghana (3, 1) and BRVM (1, 1) are the autoregressive terms significant at the 90% confidence level. However generally the coefficients on the lagged error and standard deviation terms within the GARCH conditional volatility process are considerably less than unity implying that underlying structural breaks and associated upward bias and inefficiently estimated parameters is not a problem in this case (see an extended discussion on Integrated GARCH models in Mikosch and Starica (2004); Wong and Li (2001); Lanne and Saikkonen (2003)).

Table 12

Table 10 provides evidence of the portfolio performance characteristics for the conditional models using GARCH and three-factor CAPM to model conditional means and variance-covariance matrices. Optimal asset weights are provided in Table 11. The results from the GARCH models are similar to those of the unconditional model with the Francophone portfolio having a higher Sharpe ratio (0.4226) than that of its Anglophone counterpart (0.2504). There is a greater emphasis on holding London and Nigerian assets as opposed to Ghanaian assets in the unconditional case within the Anglophone portfolio. In the Francophone case all assets are held on an approximately equal basis with notably increased holdings in BRVM in contrast to those of the unconditional model. Similar to the unconditional model the portfolio combination with all markets including BRVM and Ghana has the highest Sharpe ratio (0.4563) while this portfolio has the majority of its asset weight distributed between London, Morocco and Tunisia with Ghana and BRVM generally being held in proportions of under 10%. Owing to the inability of Ghana and BRVM to be modelled effectively using the three-factor CAPM model due to the extreme levels of price rigidity, or illiquidity, the single portfolio constructed using this model contains all markets except these two. Interestingly the Sharpe ratio obtained from using this CAPM conditional modelling technique (1.7249) is considerably higher than those of either the unconditional stochastic drift or conditional GARCH models. In line with this figure the annual mean returns (14.38%) are only slightly lower than those obtained from the other strategies while

levels of risk, or standard deviation, (1.46%) are dramatically lower. The high Sharpe ratios indicate that this conditional model infers the best risk-return related performance.

Further confirmation of the performance characteristics of the various combinations and portfolios can be seen from the loci of the investment frontiers (Figure 1). While the frontiers of the Anglophone portfolio (Figure 1(a)) fit tightly together with little real opportunity for increasing potential returns the Francophone frontiers (Figure 1(b)) have a similar profile of horizontal risk but augmented with a wider vertical distribution inferring greater opportunities to vary returns without increasing levels of risk. Interestingly the frontier loci of the aggregate portfolio excluding Ghana and BRVM (Figure 1(c)) have greater horizontal dispersion and are less spread vertically than fort he portfolio including BRVM and Ghana (Figure 1(d)). This provides further evidence that there are benefits for investment managers from the selective inclusion of BRVM and Ghanaian assets in portfolios. Figure 1(e) provides a contrasting frontier loci derived from having used the three-factor CAPM for the overall portfolio excluding BRVM and Ghana. In comparing the profile of the loci to that of Figure 1(c), that used the GARCH model, the frontier has a far greater vertical spread of potential mean return values while benefitting from a significantly decreased horizontal dispersion of values. This provides further evidence of the benefits of the three-factor CAPM in modelling equity returns.

Figure 1

5. CONCLUSIONS

This study proposes a size and liquidity augmented capital asset pricing model to explain the cross section of expected returns in the emerging market sub region of West Africa which has previously excluded from empirical valuation model research. Three West African and two North African markets are used in addition to London and Paris. The African markets are the large Nigerian stock exchange, the smaller integrated Francophone regional market of BRVM, the small Ghanaian stock exchange and the Moroccan and Tunisian bourses. Illiquidity series were constructed on a time-series cross-section basis and augment the Fama and French (1993) risk-adjusted CAPM. While costs of equity for the markets are estimated from the three-factor CAPM the potential for portfolio investment diversification is assessed through contrasting the conditional mean and variance-covariance matrices formed by rival GARCH and simple stochastic drift (iid) methodologies.

Costs of equity are found to be highest in Nigeria followed by Tunisia and then Morocco and finally by Paris and London. These results do shed light on the depth and level of adherence to regulations within these markets and the quality of market institutions. Consequently firms in Nigeria raising capital domestically to fund industrial expansion are at a distinct disadvantage to those in North Africa or Europe. Development policy should be

focussed on the design of effective regulation and appropriate enforcement mechanisms in order to make the Nigerian market a more competitive venue for raising funds in contrast to the large and active domestic banking sector.

Three models are applied to the time series data with considerable differences in the suitability and benefits of each. In practical terms the unconditional iid strategy is questioned as it is only necessity for an investment manager to only be able to predict future trends on the basis of past activity, despite the intuitive appeal of this strategy and the logical results obtained. All modelling strategies show the best trade off between risk and return, as measured by the Sharpe ratio (portfolio excess returns divided by standard deviation), is for the portfolio containing all market assets in addition to the small and highly illiquid Ghanaian and BRVM assets. This provides some evidence that there is some potential for smaller exchanges to attract investment in providing diversification potential for overseas investment fund managers. These results merit the extension of the analysis focussed on portfolios constructed using the GARCH in the construction of portfolio efficient frontiers. The frontiers that represent investment opportunities in terms of incremental changes in the mean return and associated risk are most diffuse and have greatest horizontal dispersion of loci in the case of the portfolio containing all markets except Ghana and BRVM. In marked contrast the frontier for the same aggregate portfolio this time including the small Ghanaian and BRVM markets demonstrates a more vertically flatter profile with little horizontal spread of loci. This infers that any change of asset weights results in increases/ decreases in returns with only incremental increase in risk. Finally inspection of the frontier formed from the three-factor CAPM provides the flattest vertical profile and least horizontal inclination thereby reinforcing the results obtained through analysis of Sharpe ratios where this modelling strategy inferred the best risk-return performance.

Overall this study provides substantial evidence of the benefits of size and liquidity factors augmenting the traditional CAPM in explaining the cross section of stock returns. This provides continued support for the risk-return paradigm in investment analysis. Additional development policy inferences should be directed towards improving regulation and its enforcement in Nigeria in order to reduce the high cost of equity and render the stock market more competitive as a cheaper source of funds than the well established local banking industry. Finally there is evidence from simple portfolio analysis that there are some performance benefits in minimally holding the highly illiquid assets of the Ghanaian and BRVM stock exchanges. As such there is some potential for smaller exchanges in attracting international portfolio investment to further supplement often low levels of domestic investment.

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Table 1. Market Capitalisation and Turnover profiles, 2008

	Morocco*	Tunisia	BRVM (Cote d'Ivoire)	Ghana	Nigeria	London FTSE100	Paris CAC40
Total Number of Listed Firms	78	53	39	31	234	102	40
Market Capitalisation							
Top 1 Firm Market Capitalisation to total (%)	27.55	12.51	46.51	22.10	8.40	7.50	10.58
Top 5 Firms Market Capitalisation to total (%)	57.81	43.56	71.49	67.15	27.08	29.96	37.64
Top 10 Firms Market Capitalisation to total (%)	74.29	65.23	84.50	85.03	44.38	45.34	58.96
Top 20 Firms Market Capitalisation to total (%)	88.88	88.20	96.29	98.80	64.33	65.53	81.06
Turnover							
Top 1 Firm Turnover value to total (%)	19.42	9.69	53.95	26.25	9.00	5.72	8.98
Top 5 Firms Turnover value to total (%)	58.92	38.19	89.53	79.08	36.45	23.77	33.63
Top 10 Firms Turnover value to total (%)	78.00	61.98	95.54	93.11	52.66	41.30	51.81
Top 20 Firms Turnover value to total (%)	92.01	86.51	98.70	99.71	69.59	61.10	74.54
Sector Concentration by Market Capitalization							
Financials	42.04%	57.38%	7.13%	64.84%	59.74%	-- --	-- --
Communications	27.55%	0.31%	46.36%	0.02%	1.37%	-- --	-- --
Basic Materials	3.37%	3.89%	0.53%	0.92%	0.53%	-- --	-- --
Consumer cyclical	2.55%	12.16%	2.81%	0.60%	2.81%	-- --	-- --
Consumer non-cyclical	4.35%	8.92%	25.49%	30.23%	17.79%	-- --	-- --
Diversified	7.39%	12.51%	-- --	-- --	1.57%	-- --	-- --
Energy	1.46%	0.38%	3.08%	3.30%	10.41%	-- --	-- --
Industrial	9.93%	4.45%	12.23%	-- --	5.35%	-- --	-- --
Technology	0.14%	-- --	-- --	0.08%	0.02%	-- --	-- --
Utilities	1.23%	-- --	2.37%	-- --	-- --	-- --	-- --

Source: Compiled by authors from Bloomberg

Notes: (1) *Refers to Central Market and Block Trading Market

Table 2. Market Microstructure characteristics for BRVM antennae de bourse in Mali

Investors	Variable	Order Type	2005	2006	2007	2008
Individual	Mean trade size for Equities	Purchase	558 [20]	1,173 [47]	11,263 [18]	5,800 [15]
		Sale	350 [19]	80 [12]	5,342 [30]	24,855 [26]
	Mean trade size for Bonds	Purchase	0 [0]	1,490 [2]	0 [0]	1,000 [1]
		Sale	146 [11]	80 [1]	542 [3]	167 [4]
	Number Individual investors		21	11	12	12
	Proportion of activity by single most active individual		37.58%	93.27%	98.09%	87.51%
Institutions	Mean trade size for Equities	Purchase	0 [0]	69 [2]	2,226 [9]	9,150 [11]
		Sale	0 [0]	0 [0]	260 [2]	22,294 [7]
	Mean trade size for Bonds	Purchase	431 [18]	187 [3]	503 [3]	0 [0]
		Sale	300 [1]	400 [1]	35 [1]	34 [1]
Total Individuals	Number of Certificates		19,428 [50]	59,141 [62]	10,330 [49]	710,030 [45]
Total Institutions	Number of Certificates		8,060 [19]	1,097 [6]	22,102 [15]	256,741 [19]
Total	Total traded Equities	Purchase	11,163 [20]	55,256 [49]	222,765 [27]	187,647 [26]
		Sale	6,655 [19]	962 [12]	5,862 [32]	777,423 [32]
	Total traded Bonds	Purchase	7,760 [18]	3,460 [4]	1,510 [3]	1,000 [1]
		Sale	1,910 [12]	560 [3]	2,745 [6]	701 [5]
	Traded value Equities (Mali) UK£(m)		0.46	0.29	0.41	0.47
	Traded value Equities (BRVM) UK£(m)		16.67	51.03	79.36	150.43
	% of total traded value on BRVM		2.76%	0.56%	0.52%	0.31%

Source: Compiled by authors from Societe de Gestion et d'Intermédiation (SGI), Bamako, Mali

Notes: (1) Square brackets indicate the number of matched trades undertaken on annual basis
(2) Bonds includes both Malian and UMEAO sovereign treasury and corporate instruments
(3) All investors are Malian and are either resident in Mali or France
(4) Traded Value denominated in millions GBP (UK£)

Table 3. Summary Statistics

Six sample group markets comprising of five emerging West African markets (Morocco, Tunisia, Cote d'Ivoire, Ghana and Nigeria) and two developed OECD markets (UK and France). Datastream provides the daily prices, volume and market capitalization information. Start refers to the beginning date of the daily security return data up to the final year 2007. Price is the average of daily prices over each month and is stated in domestic currency and converted to £UK using the average exchange rate for each month and country. Volume is the average of the daily trading volume over each month and is stated in thousands. Market capitalization is measured as of 1 January for each country and is equity market value for each firm expressed in millions of local currency or £UK. The bid-ask spread is generated through $Quoted\ spread_M = 1/2\left[\left(\frac{(Ask_M - Bid_M)}{(Ask_M + Bid_M)/2}\right) + \left(\frac{(Ask_{M-1} - Bid_{M-1})}{(Ask_{M-1} + Bid_{M-1})/2}\right)\right]$ applied to respective monthly bid and ask prices for individual stocks. The monthly average is taken across all stocks to obtain a market wide measure, for which an annual mean is calculated. The £UK market capitalization is derived using the end of month exchange rate for each country and month. Square parentheses indicate median values for each variable.

				Local market			£UK equivalent		
Country	**Start**	**No. Firms by trading activity**	**Zero Return (%)**	**Price**	**Volume (thousands)**	**Market Capitalization (millions)**	**Price**	**Market Capitalization (millions)**	**Bid-Ask spread (%)**
Europe									
London (FTSE 100)	1991	101	12.04 [11.93]	537.89 [539.20]	193,989.62 [141,919.17]	9,275.19 [9,959.53]	537.89 [539.20]	9,275.19 [9,959.53]	0.0085 [0.0089]
Paris (CAC 40)	1991	40	7.13 [6.26]	37.37 [35.52]	635.78 [542.28]	8,793.01 [5,342.61]	25.66 [24.59]	8,817.09 [8,322.69]	0.0042 [0.0039]
Africa									
Morocco	1993	Top 10	43.82 [39.55]	663.27 [636.16]	4,568.47 [1,507.30]	12,228.62 [8,519.74]	42.63 [40.36]	785.21 [528.93]	0.0017 [0.0000]
		40	66.43 [66.67]	616.07 [635.06]	7,071.77 [5,205.65]	4,045.14 [3,077.26]	39.62 [39.75]	263.65 [200.44]	0.0054 [0.0000]
Tunisia	1991	Top 10	51.33 [50.44]	36.42 [33.20]	648.06 [457.64]	187.80 [187.01]	20.21 [17.57]	98.03 [91.49]	0.185 [0.1977]
		37	64.97 [66.04]	34.65 [32.02]	1,084.96 [835.74]	58.59 [48.28]	18.93 [16.02]	103.70 [95.10]	0.5337 [0.6913]
BRVM (Cote d'Ivoire)	1998	Top 5	80.23 [81.91]	21,739.91 [15,113.65]	95.55 [24.94]	141,792.17 [81,624.38]	23.23 [15.52]	151.69 [83.62]	0.101 [0.0939]
		Top 10	83.86 [85.45]	27,191.23 [22,992.42]	110.87 [34.42]	89,600.37 [52,242.39]	28.54 [23.49]	95.42 [54.56]	0.0857 [0.0817]
		33	89.65 [90.47]	21,727.78 [19,088.84]	131.38 [62.19]	35,810.08 [27,313.47]	22.59 [18.93]	37.85 [27.51]	0.0592 [0.0589]
Ghana	2000	Top 5	72.24 [74.66]	2.8144 [2.8816]	945.31 [266.88]	96.77 [104.59]	1.6678 [1.8004]	56.30 [65.95]	-- --
		Top 10	74.43 [79.11]	1.6894 [1.943]	1,784.12 [772.95]	67.92 [77.29]	0.9908 [1.1474]	39.73 [47.99]	-- --
		18	77.22 [83.02]	1.0626 [1.2458]	2,565.28 [1,411.05]	41.90 [51.09]	0.622 [0.7413]	24.57 [30.53]	-- --

Nigeria	2002	Top 10	41.96 [43.18]	12.0450 [7.2429]	1,537,138.03 [920,356.09]	123,079.19 [51,593.87]	0.0518 [0.0297]	540.25 [214.07]	-- --
		Top 20	42.61 [46.47]	23.3492 [18.3349]	1,873,151.36 [1,048,748.96]	89,878.16 [52,539.05]	0.1013 [0.0772]	391.76 [217.99]	-- --
		129	63.67 [66.94]	18.7603 [18.4429]	2,631,207.67 [1,373,235.44]	30,129.70 [25,008.37]	0.0813 [0.0768]	132.49 [102.05]	-- --

Source: Compiled by authors from Bloomberg, Datastream and National stock exchanges

Table 4. Spearman's Rank Correlations

This table reports the descriptive statistics and Spearman rank correlation for the main variables employed in this study on a country by country basis. In line with Stoll (2000) the natural logarithms are taken of the variables for price, market capitalisation and volume, while volatility remains untransformed and is the monthly average of daily price variance. Price is the average of daily prices over each month and is stated in local currency units. Volume is the average of the daily trading volume over each month and is stated in thousands. MV or market capitalization is measured as of 1 January for each country and is equity market value for each firm expressed in millions of local currency units. Three liquidity measurement variables are presented. Amihud is the liquidity measure of Amihud (2002), which is defined as the daily ratio of the absolute return on a day to the UK£ trading volume for that particular day averaged over the past 1 month and provides a measure of the price impact. Turnover is a ratio of the traded volume of shares in relation to total number of shares outstanding and is scaled by the number of trading days in the month of measurement. It provides a measure of trading frequency. The final measure is the Bid Ask spread which is the average daily relative bid ask spread over the prior 1 month, where daily relative spread is the UK£ denominated spread divided by average of Bid and Ask prices. At the end of each month for the maximum period of data availability for each country cross sectional averages for each variable are calculated over the stocks in each respective market, namely Morocco, Tunisia, Cote d'Ivoire (BRVM), London and Paris. Ghana and Nigeria are omitted owing to lack of available data. Likewise at the end of each month the cross sectional Spearman's rank correlation are computed and the time series average of those correlations are reported.

Panel A: Results for London (FTSE 100 constituent companies) (1991M01 – 2007M12)

	Price	Volatility	Volume	MV	Amihud	Turnover	Bid Ask Spread
Price	100.00%						
Volatility	-14.45%	100.00%					
Volume	-26.55%	13.49%	100.00%				
MV	40.19%	-9.99%	**66.10%**	100.00%			
Amihud	-39.31%	17.41%	**-69.00%**	**-92.73%**	100.00%		
Turnover	-38.87%	23.00%	33.62%	-17.81%	2.89%	100.00%	
Bid Ask Spread	-46.32%	22.80%	**-50.10%**	**-82.25%**	**84.37%**	8.93%	100.00%

Panel B: Results for Paris (CAC 40 constituent companies) (1992M01 – 2007M12)

	Price	Volatility	Volume	MV	Amihud	Turnover	Bid Ask Spread
Price	100.00%						
Volatility	-9.54%	100.00%					
Volume	22.73%	-4.45%	100.00%				
MV	39.67%	1.75%	**66.04%**	100.00%			
Amihud	-26.57%	27.12%	**-91.69%**	**-62.36%**	100.00%		
Turnover	-23.78%	-3.52%	26.64%	-43.51%	-22.10%	100.00%	
Bid Ask Spread	-18.50%	18.26%	**-70.19%**	-49.86%	**70.76%**	-17.12%	100.00%

Panel C: Results for Morocco (1993M08 – 2007M12)

	Price	Volatility	Volume	MV	Amihud	Turnover	Bid Ask Spread
Price	100.00%						
Volatility	-9.09%	100.00%					
Volume	-28.21%	-5.47%	100.00%				
MV	25.13%	-11.47%	**54.43%**	100.00%			
Amihud	-35.62%	**55.65%**	-8.75%	-24.72%	100.00%		
Turnover	-33.92%	-4.60%	**64.75%**	-4.45%	-4.73%	100.00%	
Bid Ask Spread	6.33%	5.93%	-17.23%	-14.34%	3.09%	-13.12%	100.00%

Panel D: Results for Tunisia (1993M08 – 2007M12)							
	Price	Volatility	Volume	MV	Amihud	Turnover	Bid Ask Spread
Price	100.00%						
Volatility	-24.37%	100.00%					
Volume	-9.69%	13.82%	100.00%				
MV	47.03%	-18.99%	**50.16%**	100.00%			
Amihud	-25.36%	38.70%	**-52.09%**	**-53.46%**	100.00%		
Turnover	-1.10%	20.58%	**81.16%**	13.33%	-38.69%	100.00%	
Bid Ask Spread	-6.09%	22.02%	-40.24%	-38.40%	44.27%	-26.42%	100.00%
Panel E: Results for BRVM Cote d'Ivoire (1995M10 – 2005M03)							
	Price	Volatility	Volume	MV	Amihud	Turnover	Bid Ask Spread
Price	100.00%						
Volatility	-8.18%	100.00%					
Volume	2.32%	24.90%	100.00%				
MV	**61.39%**	3.16%	41.74%	100.00%			
Amihud	-32.45%	**67.02%**	-8.87%	-25.99%	100.00%		
Turnover	10.06%	20.26%	**73.90%**	8.59%	-9.34%	100.00%	
Bid Ask Spread	-23.79%	7.57%	1.56%	-13.13%	8.18%	0.57%	100.00%

Table 5. Total costs on liquidity proxies and measures

The results of the panel regression tests are based on a firm-monthly basis using bid-ask spread plus commission as the dependent variable. Three liquidity measurement variables are presented. Amihud is the liquidity measure of Amihud (2002), which is defined as the daily ratio of the absolute return on a day to the UK£ trading volume for that particular day averaged over the past 1 month and provides a measure of the price impact. Turnover is a ratio of the traded volume of shares in relation to total number of shares outstanding and is scaled by the number of trading days in the month of measurement. It provides a measure of trading frequency. The final measure is the Bid Ask spread which is the average daily relative bid ask spread over the prior 1 month, where daily relative spread is the UK£ denominated spread divided by average of Bid and Ask prices. Firm size is determined from the first day of each month. Volatility is the average daily stock return variance and price and volume measure the average price (local currency units) and trading volume over an annual trading period. Turnover, price, volume, and market capitalisation are all log scaled in line with Stoll (2000). N is the sample size in firm months. The White cross-section t-statistics are in parentheses.

Market	N	Intercept	Price	Volatility	Volume	Size	Amihud	Turnover	Adj-R^2
London		**0.1420**	**-0.0086**	**0.1808**	**-0.0075**	**-0.0057**			68.15
FTSE100		**[16.06]**	**[-11.66]**	**[8.54]**	**[-12.94]**	**[-7.41]**			
		0.1038	**-0.0070**	**0.1356**	**-0.0047**	**-0.0045**	**4.5977**		74.12
		[16.99]	**[-10.19]**	**[8.0945]**	**[-10.42]**	**[-6.88]**	**[8.08]**		
		0.1279	**-0.0043**	**0.1634**	**-0.0024**	**-0.0109**		**-0.0053**	69.12
		[16.27]	**[-4.65]**	**[8.15]**	**[-3.66]**	**[-9.21]**		**[-10.41]**	
		0.0917	**-0.0032**	**0.1205**	-0.0002	**-0.0092**	**4.5296**	**-0.0047**	74.91
		[17.03]	**[-3.51]**	**[7.62]**	[-0.25]	**[-8.92]**	**[8.12]**	**[-10.29]**	
Paris		**0.0587**	**0.0106**	**0.1628**	**-0.0145**	0.0007			63.97
CAC40		**[13.08]**	**[7.83]**	**[9.51]**	**[-17.35]**	[1.49]			
		0.0549	**0.0096**	**0.0778**	**-0.0132**	0.0006	**0.0007**		65.77
		[11.85]	**[6.55]**	**[2.07]**	**[-14.18]**	[1.37]	**[3.21]**		
		0.0405	**0.0107**	**0.1585**	**-0.0252**	**0.0112**		**0.0106**	64.09
		[5.61]	**[7.89]**	**[9.45]**	**[-7.12]**	**[3.32]**		**[3.15]**	
		0.0375	**0.0096**	**0.0741**	**-0.0232**	**0.0106**	**0.0007**	**0.0100**	65.88
		[5.42]	**[6.59]**	**[2.00]**	**[-6.02]**	**[3.04]**	**[3.17]**	**[2.89]**	
Morocco		**0.0375**	-0.0030	0.0405	**-0.0082**	0.0035			32.98
		[6.66]	[-1.15]	[1.22]	**[-7.82]**	[1.32]			
		0.0424	-0.0041	0.0501	**-0.0087**	0.0036	**-0.0001**		33.15
		[7.47]	[-1.62]	[1.44]	**[-8.09]**	[1.37]	**[-5.06]**		
		0.0509	-0.0045	0.0405	**-0.0103**	0.0054		0.0019	32.98
		[2.47]	[-1.27]	[1.23]	**[-3.44]**	[1.53]		[0.73]	
		0.0499	-0.0050	0.0499	**-0.0099**	0.0047	**-0.0001**	0.0011	33.14
		[2.36]	[-1.40]	[1.44]	**[-3.21]**	[1.29]	**[-4.82]**	[0.39]	
Tunisia		0.0521	-0.0101	0.1249	-0.0023	-0.0011			8.99
		[0.58]	[-0.78]	[1.06]	[-1.53]	[-0.08]			
		0.0428	-0.0117	0.1428	-0.0025	0.0004	-7.13E-07		9.01
		[0.48]	[-0.93]	[1.22]	[-1.67]	[0.04]	[-1.73]		
		0.2670	**-0.0736**	0.1537	**-0.0822**	0.0620		**0.0798**	9.26
		[2.67]	**[-2.15]**	[1.31]	**[-2.36]**	[1.81]		**[2.31]**	
		0.2747	**-0.0822**	0.1815	**-0.0903**	**0.0704**	**-9.96E-07**	**0.0875**	9.33
		[2.69]	**[-2.39]**	[1.56]	**[-2.54]**	**[2.05]**	**[-2.23]**	**[2.47]**	
BRVM		**0.3056**	-0.0440	-0.0123	-0.0052	-0.0043			17.66
		[3.21]	[-1.47]	[-0.07]	[-1.27]	[-0.18]			
		0.3143	-0.0455	0.0500	-0.0061	-0.0042	**-0.0103**		17.80
		[3.28]	[-1.53]	[0.29]	[-1.49]	[-0.17]	**[-1.87]**		
		1.1929	-0.1969	-0.0121	-0.1895	0.1486		0.1832	17.93
		[2.35]	[-1.77]	[-0.06]	[-1.78]	[1.42]		[1.71]	
		1.2260	-0.2026	0.0528	-0.1956	0.1528	**-0.0108**	0.1882	18.10
		[2.43]	[-1.83]	[0.31]	[-1.85]	[1.47]	**[-1.95]**	[1.77]	

Table 6. Summary statistics for equally weighted monthly excess returns on 9 portfolios formed on size and illiquidity for period 2002 to 2008

For each year, t, every company is ranked by its market capitalisation of equity and the end of December in year t. Stocks are then classified into 3 portfolios based on market value, from the smallest to the largest. For each size portfolio, stocks are further sorted into 3 Illiquidity portfolios based on individual stocks Illiquidity ranking in ascending order. Nine size-illiquidity are so formed and rebalanced annually. The equally weighted monthly returns on portfolios are computed each month from January to the following December. Repeating this procedure for every year results in an overall sample set of 85qually weighted monthly returns from January 2002 to December 2008. Additionally for each sample time period three zero cost portfolios, SMB(ILLIQ) representing long small size (high illiquidity) portfolios and short large size (low illiquidity) portfolios. For each year, t, every company is ranked by its market capitalisation of equity and the end of December in year t. Stocks are then classified into 3 portfolios based on market value, from the smallest to the largest. For each size portfolio, stocks are further sorted into 3 Illiquidity portfolios based on individual stocks Illiquidity ranking in ascending order. Nine size-illiquidity are so formed and rebalanced annually. The equally weighted monthly returns on portfolios are computed each month from January to the following December. Repeating this procedure for every year results in an overall sample set of 143 equally weighted monthly returns from January 1996 to December 2007.

Portfolio	S/L	S/M	S/H	M/L	M/M	M/H	B/L	B/M	B/H
Panel A: Summary Statistics for portfolios									
Mean	0.01213	0.02523	0.01862	0.02396	0.01335	0.00297	0.00191	0.00070	0.00066
Standard Deviation	0.03651	0.06455	0.08392	0.04819	0.05546	0.05129	0.07054	0.04735	0.04199
Skewness	0.45651	1.00761	3.55374	0.77642	-1.20288	-0.60386	-0.49401	-1.02858	-1.06101
Excess Kurtosis	3.48194	3.82308	21.00590	5.78562	5.39487	4.03966	5.10357	4.53671	4.27818
Panel 1A: Number stocks by market									
London	0.0000	0.0000	0.0000	25.0119	18.8333	0.1429	27.2024	23.1905	0.0000
Paris	0.0000	0.0000	0.0000	0.0000	3.2857	2.0000	0.0000	5.4286	26.6190
Morocco	2.2024	10.2738	9.2857	1.0000	0.0000	13.4881	0.0000	0.0000	0.5714
Tunisia	5.2857	5.0000	18.1429	0.7143	0.0000	2.8571	0.0000	0.0000	0.0000
Nigeria	16.5952	12.4048	1.2381	0.2619	5.9167	10.0833	0.0000	0.0000	0.0000
Overall Mean	**24.083**	**27.679**	**28.667**	**26.988**	**28.036**	**28.571**	**27.202**	**28.619**	**27.190**

Panel B: Summary Statistics for country portfolios

	London	Paris	Morocco	Tunisia	Nigeria	Ghana	BRVM
Mean	0.00281	0.00319	0.01695	0.00877	0.03153	0.02951	0.02267
Standard Deviation	0.04879	0.06445	0.04056	0.03295	0.09334	0.06524	0.06086
Skewness	-1.00684	-0.53234	0.37558	0.22266	1.49233	1.43855	0.06734
Excess Kurtosis	4.28118	5.14008	3.33823	2.84329	6.40853	5.41699	4.23087

Panel C: Summary Statistics for valuation Factors

	SMB	ILLIQ	MARKET
Mean	0.04907	0.01211	0.01100
Standard Deviation	0.19221	0.09311	0.03853
Skewness	0.85583	-2.03492	-0.68578
Excess Kurtosis	4.29265	14.70336	4.41983

Panel D: Correlations for valuation Factors

	SMB	ILLIQ	MARKET
SMB	1.0000	-- --	-- --
ILLIQ	-0.3791	1.0000	-- --
MARKET	-0.1817	-0.1532	1.0000

Table 7. Time series regressions using equally weighted monthly contemporaneous market excess returns for 9 portfolios formed on size and illiquidity for period: 2002 – 2008, for all sample markets.

Performance characteristics of one factor CAPM and four factor adjusted CAPM are compared. Stocks are sorted in ascending order on monthly basis in accordance to Amihud (2002) illiquidity measure. The size-liquidity portfolios are formed by first sorting stocks into three portfolios, Big, Medium and Small, depending on their market capitalisation and then further sorting stocks within each size portfolio into three further illiquidity portfolios, High, Medium and Low, on an annual basis. The SMB factor is formed through the difference between small and big size portfolios. Correspondingly the HML illiquidity factor portfolio is formed from the sum of the three high illiquidity portfolios less the sum of the three low illiquidity portfolios. Panel A presents parameter estimates of the capital asset pricing model, CAPM:

$$r_{it} - r_{ft} = \alpha_i + \beta_i (r_{mt} - r_{ft}) + \varepsilon_{it}$$

and panel B presents parameter estimates of the four factor adjusted CAPM model:

$$r_{it} - r_{ft} = \alpha_i + \beta_i (r_{mt} - r_{ft}) + s_i SMB_t + h_i HML_t + \varepsilon_{it}$$

where r_{it} is the return of portfolio i in month t, r_{ft} is the one month T-bill risk free rate for month t, which is taken as the one month UK Gilt rate in this case. Numbers in parentheses are t-statistics.

Portfolio	**S/L**	**S/M**	**S/H**	**M/L**	**M/M**	**M/H**	**B/L**	**B/M**	**B/H**
Panel A: CAPM-adjusted performance									
$\hat{\alpha}$(%)	0.005746	0.015440	0.007315	-0.008956	-0.000628	0.015024	-0.008905	-0.010555	-0.014679
	(0.815234)	(2.658365)	(2.198097)	(-2.659676)	(-0.200829)	(5.714534)	(-3.063927)	(-3.783402)	(-4.018000)
$\hat{\beta}$	1.170828	0.890482	0.437400	1.084511	1.271099	0.812674	0.869481	1.023881	1.508234
	(2.407435)	(4.019663)	(4.789765)	(7.551985)	(12.02806)	(9.044675)	(8.153537)	(7.728347)	(7.481895)
Adj R^2 (1)	**0.280296**	**0.273706**	**0.203446**	**0.659439**	**0.776936**	**0.415015**	**0.632022**	**0.690413**	**0.674606**
Panel B: Three-factor CAPM performance									
$\hat{\alpha}$	-0.003986	1.17E-05	-0.000718	0.000822	0.005482	0.006789	-0.000206	-0.002205	-0.005861
	(-1.612461)	(0.005040)	(-0.277460)	(0.482526)	(2.131933)	(2.460673)	(-0.172636)	(-1.608428)	(-2.421512)
$\hat{\beta}$	1.281252	1.132950	0.588637	0.925234	1.165619	0.967288	0.726537	0.891387	1.381008
	(13.42079)	(12.64468)	(9.372577)	(15.95807)	(14.40625)	(10.04871)	(35.08630)	(34.61333)	(23.20432)
$\hat{s}$	0.252594	0.248805	0.073340	-0.145098	-0.077275	0.076159	-0.126278	-0.131791	-0.167736
	(13.51976)	(13.58372)	(5.143126)	(-11.94903)	(-4.129436)	(5.272081)	(-19.10826)	(-16.72930)	(-9.098796)
$\hat{h}$	-0.320285	0.045658	0.228910	-0.074907	-0.095680	0.231128	-0.076876	-0.035149	0.067101
	(-6.716021)	(0.893526)	(5.409172)	(-5.501699)	(-2.220931)	(3.686749)	(-5.210669)	(-2.351062)	(1.847332)
Adj R^2 (4)	**0.899722**	**0.769359**	**0.488001**	**0.903812**	**0.833123**	**0.580411**	**0.904274**	**0.943464**	**0.918159**

Table 8 Pooled cross-section regression for equally weighted monthly excess returns on country portfolios with size and illiquidity for 1996 to 2007

Performance characteristics on four-factor CAPM: $r_{it} - r_{ft} = \alpha_i + \beta_i (r_{mt} - r_{ft}) + s_i SMB_t + h_i HML_t + \varepsilon_{it}$

where r_{it} is the return of portfolio i in month t, r_{ft} is the one month T-bill risk free rate for month t, which is taken as the one month UK Gilt rate in this case. Numbers in parentheses are Newey-West HAC covariance adjusted t-statistics.

Explanatory Variables	$\hat{\alpha}$	**T($\hat{\alpha}$)**	$\hat{\beta}$	**T($\hat{\beta}$)**	$\hat{s}$	**T($\hat{s}$)**	$\hat{h}$	**T($\hat{h}$)**	**Adj R^2**
Panel A: London (FTSE100 constituents)									
Excess Market alone	-0.008895	-2.943946	1.068551	8.674248					0.717251
Excess Market and SMB	-0.001676	-1.507882	0.957707	30.95860	-0.122264	-14.19419			0.946447
Excess Market and ILLIQ	-0.009538	-2.838420	1.083272	11.13293			0.039763	0.848733	0.719592
***All Three Factors**	**0.000562**	**0.508055**	**0.913666**	**32.40147**	**-0.138440**	**-17.05890**	**-0.079348**	**-7.985708**	**0.965165**
Panel B: Paris (CAC40 constituents)									
Excess Market alone	-0.012019	-3.526205	1.360263	7.126776					0.664337
Excess Market and SMB	-0.002011	-0.969315	1.206584	20.71307	-0.169512	-10.70553			0.916176
Excess Market and ILLIQ	-0.015058	-4.250473	1.429800	11.74086			0.187826	4.748031	0.734662
***All Three Factors**	**-0.003448**	**-1.670137**	**1.234842**	**20.49864**	**-0.159133**	**-9.408696**	**0.050912**	**1.809513**	**0.919697**
Panel C: Morocco									
Excess Market alone	0.010736	2.974952	0.536274	5.968566					0.252362
Excess Market and SMB	0.009019	2.342322	0.562643	6.663159	0.029086	1.992988			0.262093
Excess Market and ILLIQ	0.008387	2.452261	0.590039	5.534602			0.145222	2.360203	0.355136
***All Three Factors**	**0.003194**	**1.084671**	**0.677238**	**7.564843**	**0.071176**	**3.975410**	**0.206460**	**4.990619**	**0.441876**
Panel D: Tunisia									
Excess Market alone	0.005446	1.729132	0.285136	3.270140					0.101411
Excess Market and SMB	0.004546	1.308204	0.298961	2.815506	0.015250	0.649942			0.098235
Excess Market and ILLIQ	0.003338	1.133467	0.333384	5.031232			0.130323	3.983848	0.227331
***All Three Factors**	**-0.000362**	**-0.131396**	**0.395515**	**6.328490**	**0.050714**	**3.108864**	**0.173956**	**4.677132**	**0.290778**
Panel E: Nigeria									
Excess Market alone	0.015831	1.989239	1.406021	3.463807					0.332231
Excess Market and SMB	-0.004627	-0.954061	1.718906	15.47376	0.343276	18.58515			0.824946
Excess Market and ILLIQ	0.023389	3.127866	1.239744	5.787225			-0.437872	-5.238906	0.515334
***All Three Factors**	**0.000330**	**0.077849**	**1.623625**	**14.51152**	**0.308786**	**14.31517**	**-0.168128**	**-2.547903**	**0.846191**

Notes: (1) * indicates models selected from which Cost of Equity are estimated

Table 9. Cost of Equity estimates derived from multi-factor regression

	Cost of Equity from regression
London (FTSE100)	4.04%
Paris (CAC40)	9.19%
Tunisia	12.53%
Morocco	18.62%
Nigeria	45.45%

Notes: (1) Annualized cost of equity estimates generated at 12/2008 from the total risk premium
(2) The UK Gilt/ Treasury rate is used in each case for risk free rate

Table 10. Performance of conditional strategies

Universe	Mean return (Annualised)	Standard Deviation (Annualised)	Maximum (Annualised)	Minimum (Annualised)	Sharpe Ratio (Annualised)
Strategy: IID					
Anglophone	21.39%	9.24%	72.95%	2.78%	0.4813
Francophone	18.38%	7.80%	44.62%	-12.31%	0.5366
Overall (incl. Ghana/ BRVM)	18.38%	6.14%	44.98%	-12.71%	**0.6792***
Overall (excl. Ghana/ BRVM)	15.62%	6.65%	46.84%	-13.15%	0.5035
Strategy: GARCH					
Anglophone	13.12%	9.38%	14.82%	10.52%	0.2504
Francophone	20.58%	10.13%	27.01%	17.26%	0.4226
Overall (incl. Ghana/ BRVM)	16.67%	7.17%	19.89%	14.50%	**0.4563***
Overall (excl. Ghana/ BRVM)	13.87%	7.69%	17.21%	9.43%	0.3312
Strategy: Conditional (Multifactor CAPM)					
Overall (excl. Ghana/ BRVM)	14.38%	1.46%	41.13%	-19.35%	**1.7249**

Notes: (1) All statistics are represented in their annualised forms.
(2) Portfolio returns and standard deviations are the annualised forms of series generated by quadratic technique of portfolio optimisation of the two underlying component series in each case.
(3) The two Sharpe ratios highlighted in bold with an asterisk are the largest for each respective portfolio combination across all models used to construct the sample means indicating that these portfolios offer the best returns in relation to risk, or standard deviation.
(4) The Sharpe ratio without an asterisk indicates the highest value across all portfolios and unconditional/ conditional strategies
(4) The annualised average Sharpe ratios are defined as the mean return in excess of the Treasury Bill rate (in this case the UK Gilt 3 Month rate), divided by the standard deviation.
Anglophone portfolio indicates inclusion of markets following English Common law, i.e. London, Nigeria and Ghana. Francophone indicates those with French civil code i.e. Paris, Morocco, Tunisia and Cote d'Ivoire (BRVM).

Table 11. Minimum variance asset weights for portfolio unconditional and conditional strategies

	2003	2004	2005	2006	2007	2008
Panel A: IID						
Panel 1: Anglophone Portfolio						
London	22.75%	85.82%	77.18%	53.20%	62.28%	41.51%
Ghana	40.94%	8.77%	7.13%	26.93%	36.51%	48.95%
Nigeria	36.31%	5.40%	15.70%	19.87%	1.21%	9.54%
Panel 2: Francophone Portfolio						
Paris	2.91%	28.95%	27.68%	7.95%	34.09%	3.65%
Morocco	10.63%	33.77%	27.37%	28.56%	6.63%	39.78%
Tunisia	68.88%	28.42%	37.97%	38.05%	40.49%	42.90%
Cote d'Ivoire (BRVM)	17.58%	8.86%	6.98%	25.43%	18.79%	13.67%
Panel 3: Overall (incl. Ghana/ BRVM) Portfolio						
London	0.76%	25.54%	46.18%	13.12%	34.58%	12.42%
Ghana	1.63%	1.33%	2.09%	12.95%	6.77%	3.00%
Nigeria	25.92%	10.58%	13.62%	18.31%	0.49%	0.70%
Paris	1.85%	0.00%	0.00%	0.06%	4.97%	0.00%
Morocco	0.61%	23.43%	14.28%	23.88%	3.76%	40.09%
Tunisia	56.01%	29.65%	17.95%	17.93%	34.36%	33.51%
Cote d'Ivoire (BRVM)	13.23%	9.47%	5.87%	13.75%	15.07%	10.29%
Panel 4: Overall (excl. Ghana/ BRVM) Portfolio						
London	0.21%	29.18%	47.92%	30.63%	53.54%	19.44%
Paris	1.73%	0.00%	0.00%	0.00%	7.26%	0.00%
Morocco	2.01%	28.56%	16.12%	18.05%	4.19%	47.69%
Tunisia	68.15%	32.21%	22.12%	29.06%	34.09%	32.75%
Nigeria	27.91%	10.06%	13.84%	22.26%	0.91%	0.13%
Panel A: GARCH						
Panel 1: Anglophone Portfolio						
London	1.59%	2.37%	19.79%	19.70%	32.47%	18.23%
Ghana	24.74%	30.40%	30.44%	24.87%	17.50%	31.52%
Nigeria	73.67%	67.23%	49.76%	55.43%	50.03%	50.24%
Panel 2: Francophone Portfolio						
Paris	1.96%	6.54%	30.55%	25.48%	27.65%	18.09%
Morocco	50.11%	50.70%	39.52%	31.71%	24.72%	39.86%
Tunisia	21.97%	19.30%	13.31%	16.10%	20.75%	19.79%
Cote d'Ivoire (BRVM)	25.95%	23.46%	16.62%	26.71%	26.88%	22.26%

Panel 3: Overall (incl. Ghana/ BRVM) Portfolio						
London	0.00%	1.84%	29.92%	32.62%	27.72%	21.16%
Ghana	13.20%	0.84%	0.00%	7.34%	20.22%	9.82%
Nigeria	28.02%	22.47%	12.41%	16.13%	1.32%	0.00%
Paris	0.04%	0.01%	0.00%	0.00%	0.00%	0.00%
Morocco	0.92%	4.83%	16.65%	14.40%	12.73%	22.80%
Tunisia	47.75%	55.34%	31.19%	21.94%	28.70%	37.42%
Cote d'Ivoire (BRVM)	10.08%	14.67%	9.83%	7.57%	9.31%	8.80%
Panel 4: Overall (excl. Ghana/ BRVM) Portfolio						
London	0.47%	7.92%	49.93%	49.62%	58.12%	45.97%
Paris	0.00%	0.00%	0.00%	0.00%	1.87%	1.70%
Morocco	11.12%	10.36%	10.17%	11.07%	0.99%	2.42%
Tunisia	43.84%	38.97%	20.38%	13.06%	20.23%	27.87%
Nigeria	44.58%	42.74%	19.53%	26.24%	18.79%	22.03%
Panel A: Linear Multifactor CAPM						
Panel 1: Overall (excl. Ghana/ BRVM) Portfolio						
London	65.75%	64.12%	66.02%	63.73%	63.45%	64.45%
Paris	26.38%	23.45%	19.94%	22.93%	24.94%	27.07%
Morocco	1.90%	3.71%	4.55%	4.11%	3.26%	2.46%
Tunisia	0.21%	0.60%	0.89%	0.61%	0.31%	0.15%
Nigeria	5.76%	8.12%	8.60%	8.62%	8.04%	5.86%

Table 12. GARCH model parameters for period January 2002 to December 2008

	Coefficient	Z-statistic	Coefficient	Z-statistic	Coefficient	Z-statistic	Coefficient	Z-statistic	Coefficient	Z-statistic	Coefficient	Z-statistic	Coefficient	Z-statistic
	London (3,2)		**Paris (1,3)**		**Morocco (1,1)**		**Tunisia (1,2)**		**Nigeria (2,1)**		**Ghana (3,1)**		**BRVM (1,1)**	
	Mean Equation													
α	0.006716	1.68	0.008618	1.50	0.014970	2.59	0.007857	2.81	0.012014	1.27	0.017108	2.35	0.018217	2.11
β	0.053461	0.46	0.065264	0.52	0.091237	0.64	-0.003713	-0.06	0.298144	2.22	0.267880	1.66	0.305561	2.44
	GARCH representation for Residuals													
ω_0	0.000374	0.81	0.000448	1.34	0.000927	0.26	0.000837	2.22	0.000109	1.29	0.000871	1.19	0.003581	1.89
δ1	0.260470	2.08	0.364725	2.26	0.041507	0.24	-0.209684	-3.96	0.235189	44.48	0.345354	0.96	0.220423	1.61
δ2	0.050526	0.37	-- --	-- --	-- --	-- --	-- --	-- --	-0.269330	-14.06	-0.213891	-0.54	-- --	-- --
δ3	0.451902	1.79	-- --	-- --	-- --	-- --	-- --	-- --	-- --	-- --	-0.068300	-0.43	-- --	-- --
δ4	-- --	-- --	-- --	-- --	-- --	-- --	-- --	-- --	-- --	-- --	-- --	-- --	-- --	-- --
δ5	-- --	-- --	-- --	-- --	-- --	-- --	-- --	-- --	-- --	-- --	-- --	-- --	-- --	-- --
γ1	-0.164518	-0.92	0.978789	4.06	0.388420	0.16	0.247752	0.54	1.027492	64.53	0.694258	2.27	-0.223282	-0.48
γ2	0.352642	1.40	-0.838014	-2.44	-- --	-- --	0.151162	0.44	-- --	-- --	-- --	-- --	-- --	-- --
γ3	-- --	-- --	0.433523	1.86	-- --	-- --	-- --	-- --	-- --	-- --	-- --	-- --	-- --	-- --
γ4	-- --	-- --	-- --	-- --	-- --	-- --	-- --	-- --	-- --	-- --	-- --	-- --	-- --	-- --
γ5	-- --	-- --	-- --	-- --	-- --	-- --	-- --	-- --	-- --	-- --	-- --	-- --	-- --	-- --

Notes: (1) GARCH lag order chosen *via* Schwartz-Bayesian Criterion informational criterion (SBC).
(2) Z-statistic significance levels are as follows: 60% confidence is 0.84; 70% confidence is 1.04; 80% confidence is 1.28; 90% confidence is 1.65; 95% confidence is 1.96; 99% confidence is 2.58.

Figure 1. Efficient Frontiers for portfolio using conditional multi-factor CAPM (Excl. Ghana and BRVM) for 2002M12 to 2008M12

Figure 1(a) GARCH Anglophone portfolio

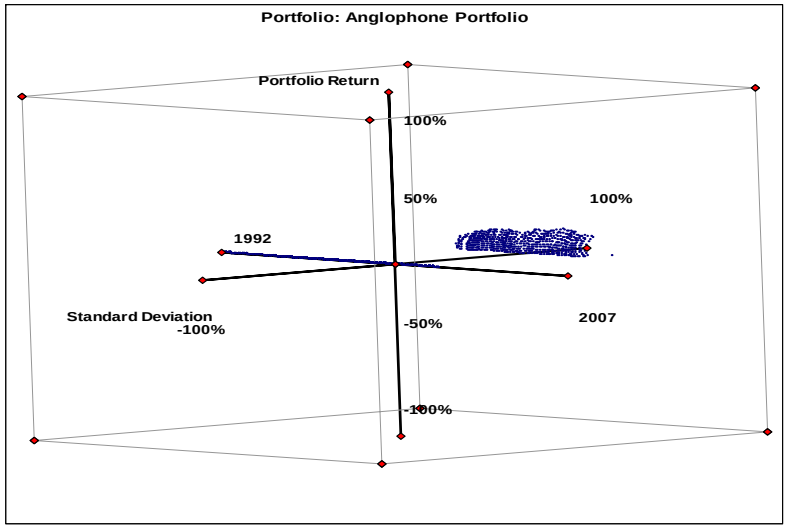


Figure 1(b) GARCH Francophone portfolio

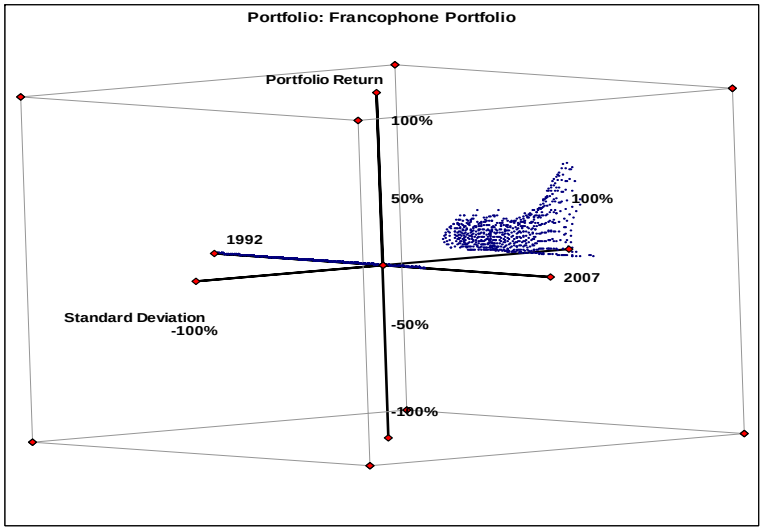


Figure 1(c) GARCH Overall (Ex. Ghana/ BRVM) portfolio

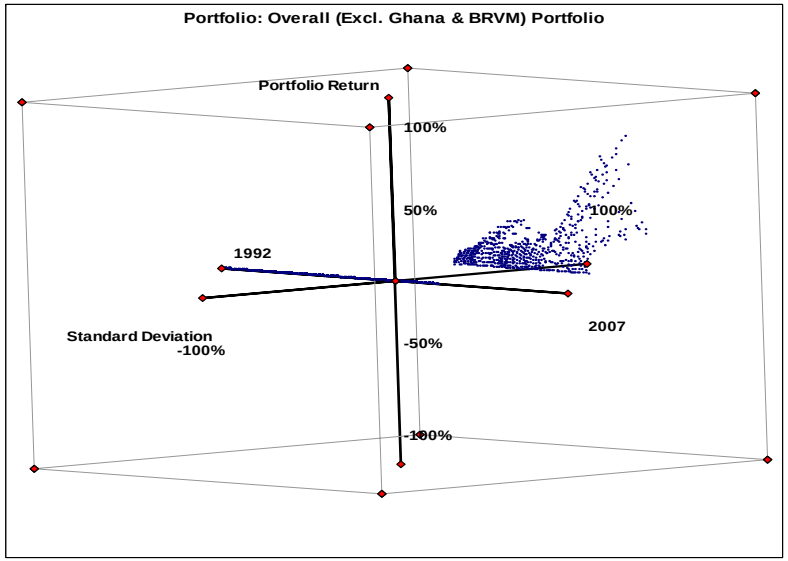


Figure 1(d) GARCH conditional (Incl. Ghana/ BRVM) portfolio

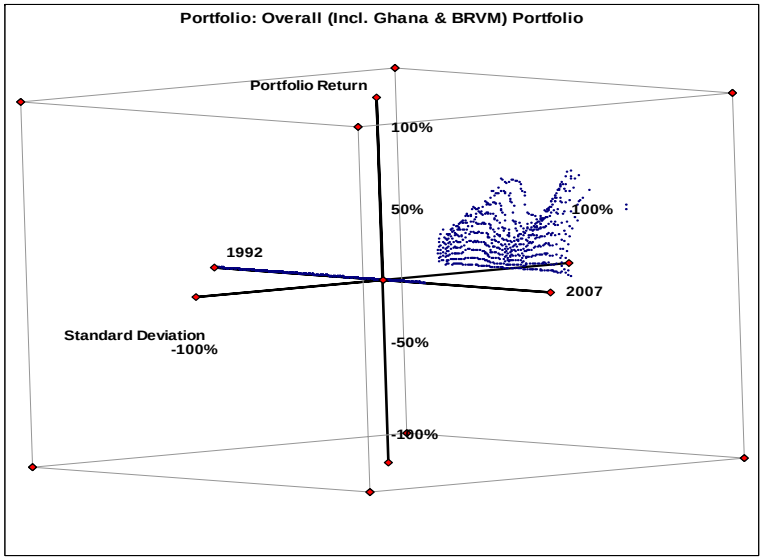
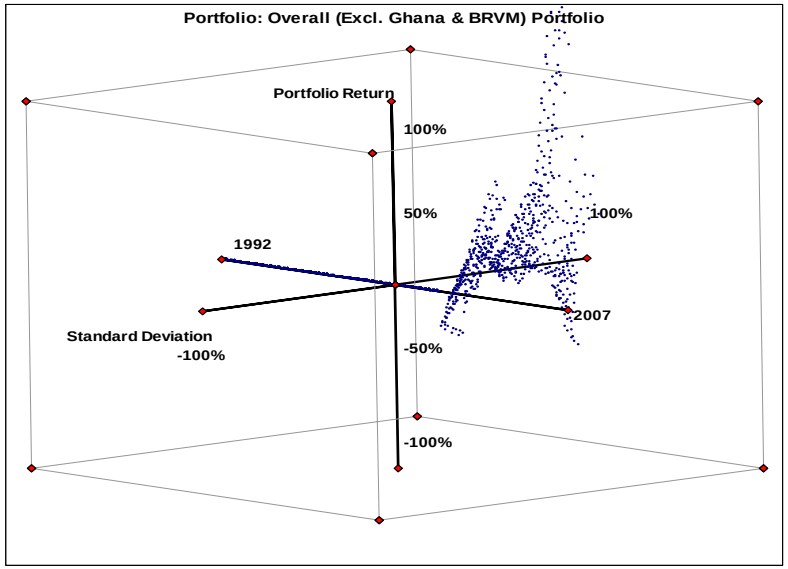


Figure 1(e) Multifactor CAPM Overall (Ex. Ghana/ BRVM) portfolio



Appendix 1 Summary of Secondary Market regulations and fees for selected countries

	Morocco	Tunisia	BRVM (Cote d'Ivoire)	Ghana	Nigeria (Lagos)
Nature of Regulator	Exchange and Central Bank perform role of market surveillance and regulator			Self Regulatory (Independent regulator)	
Commercial Law	French civil code			English common law	
No. Brokers	15	24	9 in Cote d'Ivoire; 4 in Benin; 3 in Senegal; 1 in all other UMEAO countries	16	219
Short Sales Permissible?	No	No	No	No	No
Market Clearance Procedures	Fully G30 compliant including custodial facilities. DVP undertaken T + 3.		Partial adherence to G30 standards. Custodial facilities provided by local and international brokers	Partial adherence to G30 standards. Custodial facilities provided by local and international brokers	Custodial facilities provided by brokers with sufficient capitalisation. Mostly G30 compliant and DVP undertaken at T + 3
Trade/ Clearance Guarantee Fund	Yes	Yes	Yes	Yes	Yes
Capital Gains Tax	Exempt	Exempt	Exempt	Exempt	10%
Other Taxes and Fees	VAT applied to the amount of commissions is 10%. No other tax/ fees.	None	None	10% withholding tax	Stamp Duty of 0.07%. SEC fee of 1%. VAT levied as % of commission fee. Withholding tax on dividend and interest is 10%; corporate income tax, 35%
Commission	Standard fee of 0.1% of trade value in Moroccan Dirhams (MAD) levied against buyers and sellers engaging in securities transfer or dealing. This fee, Negotiation des Titres, is applied to both buy and sell legs of trade.	Two fees: First is fee of 0.20% on transactions less than 50,000TDN and 0.10% on those above. Second fee is sliding schedule depending on value traded and ranges from 0.25% <0.5m TDN to 0.05% for >3m TDN.	Discretion of individual local brokerage firms (SGIs)	Sliding scale of fees from 2.5% for transactions less than 5m ¢ to 1% for those over 500m ¢	Sliding scale of fees from 1% to 2.75% down to 1.00% of consideration Depository settlement fee: 0.30%

Notes: The Group of Thirty is the most influential body to encourage the standardisation and improvement in global securities administration. Following a symposium in London in March 1989, the following recommendations were agreed: i) Brokers should match trades on day after deal date (T+1); ii) Trade confirmation on trade day plus 2 days (T+2); iii) Central Depository for safe keeping of shares; iv) Net basis settlement of cash and stock; v) Settlement takes place as delivery vs. payment or receipt vs. payment; vi) Settlement in same day funds; vii Settlement effected on trade date plus 3 days (T+3) 8; viii) Securities lending should be permitted; ix) International securities numbering system must be adopted (ISIN code).

Appendix 2. Portfolio Strategies: Conditional models

Following Jackson and Staunton (2003) and Harvey (1994) in a mean-variance framework (Markowitz, 1959) an investor's utility preference function can be expressed

$$U = E(r_p) = \frac{1}{2}\lambda\sigma_p^2 \qquad (10)$$

where the higher the value of the risk aversion coefficient, λ, the larger the portfolio risk that is subtracted from the portfolios expected return. At a given λ the portfolio return and variance can be expressed

Portfolio Return (conventional notation): $E(r_p) = \sum w_i E(r_i)$ (11a)

Portfolio Return (matrix notation): $w^T \mu$ (11b)

Portfolio Variance (conventional notation):

$$Var(r_p) = \sigma_p^2 = \sum\sum w_i w_j \operatorname{cov}(i, j) \quad \text{where} \quad \operatorname{cov}(i, j) = \sigma_i^2 \qquad (12a)$$

Portfolio Variance (matrix notation): $w^T V w$ (12b)

where $E(r_i)$ is the expected return for the i[th] asset, and σ_i is the risk (standard deviation of returns) for the i[th] asset. w represents N x 1 vectors of the weights of the individual assets and V is the N x N variance-covariance matrix, and N is the number of markets under consideration. In this paper, N = 2.

The investment proportions are unrestricted in size but must sum to unity. However, this does allow extremely large short and long positions in any of the markets under consideration. As thin trading is a widespread problem across emerging markets, and particularly so in the African markets a plausible constraint is that all short sales be disallowed, that is, $w_i \geq 0$ for i = 1,….N. This adds a second constraint to the mean-variance optimisation problem. All strategies are in UK£ (sterling) and assume that no currency hedging takes place. While this assumption is reasonable for developed and larger emerging markets it is not for smaller emerging markets. It is also an assumption that there is a market for suitable future and forward contracts as well as analyst coverage generating a sufficient supply of economic information. Thus, this study uses the UK£ (sterling) as the common numeraire currency[3].

The strategies evaluated in this paper involve solving (11a), subject to the global minimum of (12a) at the end of each month and holding the implied portfolio for the next

[3] Harvey (1993b) analyses the international asset allocation problem and makes the assertion that portfolio selection should include currency portfolios in the form of local deposits or loans. The solution to the quadratic programme should deliver the optimal asset allocation as well as optimal currency hedges. It is likely that this method would be preferential to models where perfect foresight of exchange rates is required, such as currency covariance augmented CAPM models as holdings of existing domestic deposits could be recursively modified within the optimisation process.

month. The sample is updated using a 12-month moving window and the portfolio is re-optimised at each point in time. In all strategies, transactions costs are ignored. In each case the minimum variance portfolio is analysed, that is, the investment weights match the weights implied by the minimum variance portfolio over the previous 12 months. These weights are used to form a portfolio that is held over the next month. As such in the mean variance optimisation problem in (11a)/ (12a), three sets of inputs are needed: means, variances and covariances. At the end of each month the investor attempts to design the portfolio that guarantees the highest possible expected return for the minimum level of volatility. Using conditional models is preferential for the mean-variance problems in terms of opening the possibility to provide the best possible forecasts of the expected returns, variances and covariances for the next period.

The conditional asset allocation implements forecasting models for the inputs of the mean-variance problems. The combination of a constant and the autoregressive term are adopted as the conditional mean for the GARCH model while a combination of fixed constant and CAPM regression betas are used for the mean equation in the multifactor CAPM. The regression models of conditional means use a number of information variables,

$$E(r_{it} \mid Z_{t-1}) = Z_{t-1}\delta_i \qquad (13)$$

where r_{it} is the return on country *i* from t–1 to t and Z_{t-1} is a 1 x *l* vector of *l* global or market specific information variables. In the multifactor CAPM this is the combination of risk premiums known at time t–1. δ_i is a *l* x 1 matrix of coefficients. The errors from these mean regressions, ε_{it}, are assumed to be unrelated to the conditioning information Z_{t-1}.

The portfolio problem also requires a forecast of the variance-covariance matrix. Consider the covariance between asset i and j:

$$\operatorname{cov}[r_{it}, r_{jt} \mid Z_{t-1}] = E[(r_{it} - E[r_{it} \mid Z_{t-1}])(r_{jt} - E[r_{jt} \mid Z_{t-1}]) \mid Z_{t-1}] \qquad (14)$$

Given the regression errors in (31) it is possible to rewrite this as

$$\operatorname{cov}[r_{it}, r_{jt} \mid Z_{t-1}] = E[\varepsilon_{it}\varepsilon_{jt} \mid Z_{t-1}] \qquad (15)$$

The conditional covariance is the forecasted value of the product of the residuals for the regression models for asset i and asset j.

In principal, the conditioning information for asset *i* and asset *j* is different. The approach in this paper follows Harvey (1994) by using the unconditional mean of the product of residuals as the forecasted variance-covariance matrix. This implicitly assumes that the product of the residuals is not predictable as in Solnik (1993). However, the matrix used here is not the unconditional variance-covariance matrix but the average conditional variance-covariance matrix where the Z_{t-1} variables are permitted to affect the means. This approach greatly simplifies the estimation.

In a similar manner to the unconditional asset allocation, the variance-covariance matrix is based on a 12 month moving window average of the product of the regression residuals throughout the sample. In the analysis the regressions are estimated over the full sample which implies that the regression coefficients, δ_i, are constant.