MARSHALLIAN MACROECONOMETRIC MODEL
A Bayesian versus Non-Bayesian Perspective

Jacques Kibambe Ngoie
Arnold Zellner
Reneé Van Eyden[♣]

Abstract

The present paper investigates the use of a disaggregated Marshallian Macro econometric Model (MMM-DA) of the South African economy in ascertaining the virtues of disaggregation that are namely: more data and information improve prediction ability of macromodels; and more data leads to more powerful tests of a broader range of hypotheses regarding parameters' values. Also more possible models in forecasting experiments can be tested. To check on the predictive performance of our MMM-DA, results of forecasting experiments are presented that show it forecasts reasonably well and better than a benchmark autoregressive, leading indicator model. This paper also provides a comparison between estimating our MMM-DA using ISUR (Iterative Seemingly Unrelated Regressions) without Bayesian techniques versus an ISUR with Bayesian techniques. The two Bayesian techniques discussed in the paper are: (1) the MCMC (Markov Chain Monte Carlo Simulations); and (2) the DMC (Direct Monte Carlo Simulations). Furthermore, this research highlights the role played by the choice of an appropriate loss function, when needed, in order to improve the model's applicability for policy guidance as well as the use of shrinkage techniques in improving predictions.

Keywords: Direct Monte Carlo Simulations; Marshallian Macroeconometric Model; Markov Chain Monte Carlo Simulations; and Shrinkage Techniques.
JEL Code: E27

[♣] Jacques Kibambe Ngoie: Department of Economics, University of Pretoria, Pretoria, South Africa (e-mail: Jacques.kibambe@up.ac.za); Arnold Zellner: HGB Alexander Distinguished Service Professor Emeritus of Economics and Statistics, Graduate School of Business, University of Chicago, Chicago, Illinois (e-mail: fazellne@chicagogsb.edu); and Reneé Van Eyden: Department of Economics, University of Pretoria, South Africa (e-mail: renee.vaneyden@up.ac.za).

INTRODUCTION

The progress and reliability of forecasting macro models for world economies have been examined in several modelling workshops. It is noticeable that the scarcity of sound forecasting frameworks weakens budgeting and planning processes in the developing regions of the world when it comes to public investment in human capital. And the lack of required expertise combined with the inexistence of a nurtured centralised data warehousing system, which both are associated with massive financial requirements, constitute an obstacle to the development of forecasting frameworks. In one of its recent report, the Economic Commission for Africa (ECA) depicted the challenge faced by African governments in their modelling exercises.

Forecasting can be perceived as a tool of guidance for policy-making units in order to achieve long-term goals. South Africa in particular has set very specific developmental goals aligned with the MDGs. These goals are mainly: (1) long term economic growth (6 – 7%); (2) poverty eradication; (3) improved health and education for the population; …. Good projections are linked to sustainability of socio-economic policies. In line with the MDGs and with continuous pressure from the IMF (International Monetary Fund), several attempts have been made to build forecasting models and some models have been successfully run. We can recall few traditional types of economy-wide models often discussed in the policy making process: (1) the IMF financial programming framework; (2) the World Bank RMSM models; (3) the 'three-gap' models or the 'two-gap' models for Africa; (4) the Computable General Equilibrium Models; (5) the Dynamic, Large-Scale models; (5) the Project Link; (6) the generalized Neo-Keynesian macro model; (7) the Dynamic Stochastic General Equilibrium model (more recently); etc.

We have noticed an extensive use of the two-gap model in African forecasting with regard to attraction of foreign direct investment needed for economic growth. Since this type of modelling process is money driven, the credit it can receive from purely academic analysts might be questionable. Nevertheless, the underpinning foundation of the two-gap model is that of Domar Growth model. It highlights the contribution of foreign resource inflows to enhance local economies.

Producing reliable macro models able to describe a country's economy with the aim to evaluate alternative policies has been the major concern of several thinkers for many years. Different schools of thought have been and are still competing around the issue. Some placed more emphasis on the key role played by money in the economy (monetarist and neo-monetarist), while others preferred to weight more emphasis on the cyclicality of economic systems (real business cycle models and generalized business cycle models). For several years, macro modellers have also made use of Keynesian principles of economic system to build models (Keynesian models and Neo-Keynesian models). The more recent literature has been enriched with several improvements obtained on the empirical performance of the three-equation New Keynesian macro model (the Dynamic Stochastic General Equilibrium Model) applied on European panel data. The improvement obtained was generated by the use of real time information under the Taylor rule[1].

The use of benchmark models has always been recommended though in order to provide a better assessment of the performance of any new model. The benchmarks models that are most often utilized are the following: (1) ARLI 3 (Autoregressive Leading Indicators of order 3); (2) the univariate ARIMA (Autoregressive Integrated Moving Average, Box-Jenkins); and (3) the VAR (Vector Autoregressive, Bayesian or Non-Bayesian).

BACKGROUND

On the basis of the two-gap model, the 'Bretton Woods' Institutions have developed the 'Revised Minimum Standard Model'. Fund's related models are meant to address balance of payment deficit problems in member states. Several attempts have been made to forecast African growth using the regressions approach. However this approach has the weakness to require the supply of future values of the exogenous variables. Forecast values of exogenous series must be obtained from a univariate framework or its multivariate counterpart VAR (Vector Autoregressive). Countries like Kenya made use of VAR to obtain a period forecast for their exchange rate while the policy impact of the key variables was captured through impulse response functions. VAR models have been extensively used in many African macroeconomic models although their outcomes are more used for policy evaluation.

[1] Paloviita M.: 'Estimating a Small DSGE Model under Rational and Measured Expectations: Some Comparisons'. Bank of Finland. Session Paper no. 14/2007.

AR (3) models including lagged leading indicators have been successfully used in point forecasting frameworks. Though, empirical results (Zellner & Tobias, 2000) have shown the improvement effects of disaggregating in both ARLI (Autoregressive Leading Indicators) as well as Marshallian Models.

This model is based on a sectoral disaggregating including demand, supply and entry/exit relations with sound consideration of labor productivity as affected by social ingredients such as health and education. Few pilot studies on comparative analysis between 'Aggregation and Disaggregation in term of forecasting performance' could be located. Zellner and Tobias (1999) published a paper that focused on a comparative analysis between Aggregated Forecasting Model and Disaggregated Forecasting Model of median growth for eighteen industrialised countries. They made use of MAEs and RMSEs results to support the hypothesis that 'disaggregation' produces better forecasting outcomes. The aggregated approach used in their paper included median rate variables obtained from all eighteen countries[2]. In their alternative way to employ a disaggregated approach, Zellner and Tobias referred to the same 'Autoregressive Leading Indicators of order 3', ARLI (3) process, while each of the estimates carried two subscripts. One subscript for the country and the other one for the year considered. The disaggregated model increased the number of estimate equations and provides higher reliability to the panel data[3]. Outcomes of their research paper suggest that disaggregation is more likely to produce better forecast than 'aggregation', although their disaggregated equations included one aggregated variable: the annual median growth of Real GDP. Other evidence of improved forecasting results could be drawn from such comparative studies especially when considering the fact that disaggregation provides more observations to estimate with reasonably better model specification.

Multiple equation forecasting models brought forward the use of: (1) single information estimation technique; (2) limited information system methods (Two Stage Least Square, Instrumental Variable Estimation, Limited Information Maximum Likelihood); and (3) full information system technique (Three Stage Least Square and Full Information Maximum

---

[2] Zellner and Tobias modelled the median growth rate of GDP (aggregative) using an AR(3)LI process that includes three lagged variables of the median growth of GDP together with two other median growth variables: the median growth rate of Real Money; and the median growth rate of Real Stock Prices.

[3] Alternatively, in their disaggregating model, Zellner and Tobias made use of ARLI relationships using the same variables as the one used in the aggregated model. They firstly allowed all coefficients to vary across countries, secondly they imposed all coefficients to be equal across countries and thirdly they imposed restriction only on leading indicators' coefficients to be the same across countries.

Likelihood) in forecasting frameworks (Challen and Hagger, 1983) using VAR forecasting approach. The MMM belongs to the group of chaotic models that generate booms and bubs as compared to the sin-waves generated by linear models. It generates non-linear differential equation systems. Using Newton's theory of motion, scientists have since elaborated extensions to the theory and later on (1975) they became aware of another kind (the third) of motion that they called 'chaos'. 'Chaotic' is the description for erratic and quasi periodic models that are found in several systems.

In the general forecasting literature as suggested for developing economies, it is important to mention the "Excel-based model for forecasting (EBMF)" developed in 2004 by Huizinga *et al* (Huizinga and Alemayehu, 2004). The EBMF was established on AD – AS framework. That model also includes sectoral differentials using the CES function although the closing of the systems differ from the marshallian approach and the output growth is obtained from aggregation of investment consumption, exports, government expenditures, etc.

THE VIRTUES OF DISAGGREGATION

The use of disaggregating process in this MMM is sustained by sector differentials that prevail in the South African economy. The output growth per sector presents disparate behaviour to such extends that using aggregate data entails loss of useful information (see fig. 1). Moreover, aggregate models are unable to analyze detailed policy shocks. Aggregate frameworks suffer from loss of crucial information and that lead to inaccurate policy recommendation without specific consideration of sectoral differentials. A major concern is then raised regarding the veracity, or rather accuracy, of the existing forecasts used for policy analysis. If sector differentials are not considered, the forecasting frameworks will remain questionable. Improvement effects of disaggregation have been captured in previous studies as measured by reduced 'Mean Absolute Errors' (MAEs) and 'Root Mean Squared Errors' (RMSEs). While using disaggregated frameworks, MAEs and RMSEs displayed smaller error figures compared to aggregate models and that is noticeable as improvement in forecasting performance. We decided to disaggregate by sector of production (industries) as each sector portrays specific characteristics. Although, labor force is most often perceived from an aggregate point of view, both: labor; capital; and technology have different functioning from one market to another. And both labor and capital evolve in markets that are different according to the sectors. The different growth rates presented in each sectors may not be that larger, however, it is important to predict the behaviour of these sectors individually. Marshall emphasized that the process of entry and exit of firms is instrumental in producing long run equilibrium. Assuming that sectors error terms are correlated across sectors (see correlation test of the panel), joint estimations with Stein-like shrinkage techniques can be combined in order to improve the predictive accuracy of estimates at both disaggregate and aggregate level. Stein-like shrinkages work reasonably well using time varying parameters to allow for possible 'structural breaks'. Theses shrinkages also have the advantage to deal with parameters that can vary with time. The synchronization in sectoral rates of growth is very weak (see Fig 1). This leads to consistent loss of information when one makes use of aggregate models. Sectors or countries exhibit both seasonal, cyclical, and trend behaviour differences that only disaggregated models can capture.

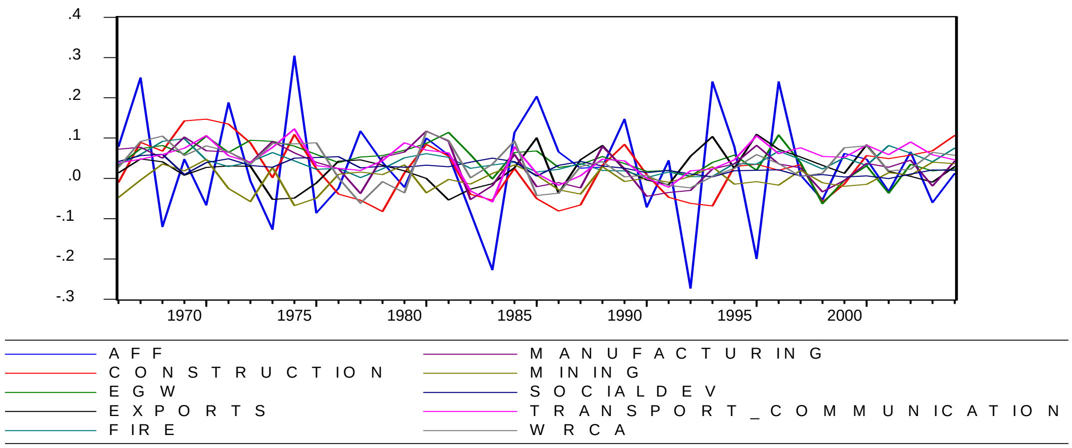

Fig. 1: Example of the South African annual real output growth rates per development sectors

With:

- AFF: Agriculture, Fishing and Forestry (Agric)
- EGW: Electricity, Gas and Water (El)
- FIRE: Financial Intermediates and Real Estate (Fin)
- WRCA: Wholesale, Retail trade, Catering and Accommodation (Whol)

MODEL SPECIFICATION

I. General characteristics of a macro econometric model

In the present research, the use of a disaggregated Marshallian Macro econometric Model (MMM-DA) is justifiable considering its ability to produce better forecast and its higher policy analysis performance. Several criteria (statistical or theoretical) may be used to assess the model's prediction[4] (forecasting) ability[5]. However, in the present study, only two statistical criteria are mostly referred to: the RMSE (Root Mean Square Error) and the MAE (Mean Average Error). However, as a theoretical assessment, we provide a thorough discussion of the estimates and the prediction results obtained. Predictions are analyzed in order to establish whether they are supportive of underpinning economic theory or not. The use of a MMM-DA provides greater ability to apply policy simulations and therefore generate the effects at both sectoral and national levels. The more disaggregated is a model; the better results are obtained when applying policy shocks.

The literature suggests some general characteristics that may be used to assess macro econometric models' validity. In various cases, the following properties need to be observed: (1) the simplicity and consistency of the model; (2) the relevance and support of the economic theory; (3) the reliability and possibility to estimate the functional form; (4) a solid stochastic specification of the model; (5) the consistency and significance of explanatory variables; (6) the appropriate number of equations as compared to the number of endogenous variables; (7) an adequate identification of parameters; and (8) an appropriate use of mathematical theory in obtaining the results. The use of simulation experiments is a very intuitive way to acquire information about dynamic properties of the model. The thorough consideration of the above-mentioned checking tips for the model's properties does not exclude the fact that all related statistical tests must be conducted cautiously. These range from the basic data evaluation to the more advanced diagnostic and forecasting tests. The larger a model becomes in terms of the number of non-linear equations used, the more difficult it is to determine whether the model carries a unique solution or not.

[4] Most often the concepts 'prediction' and 'forecasting' are considered as interchangeable. However, in this study, 'prediction' is the most appropriate concept since we make use of a structural model.
[5] Zellner A.: "Basic Issues in Econometrics", The University of Chicago Press, 1984.

Concerning the choice of asymptotically justified estimates (Zellner, 1984), the size of the finite-sample matters. Asymptotically justified estimates may induce different values in relation to the type of specification errors used. The present study does not provide any sensitive analysis that can be used in order to assess the choice of asymptotically justified estimates as it is assumed that specification errors do not affect alternative estimates differently.

II. Deriving the Marshallian Model (Disaggregated Model)

II.1. Firms optimization process

Assuming a Cobb-Douglas production function as follows:

$$Q = A_N (z L)^{\alpha} K^{\beta}$$

(2.1.1)

with: - A : Neutral technological change per sector;

- z : Level of human of human capital in per capita terms[6];

- zL : Effective Labor Input

$$Z = zL$$

(2.1.2)

Human capital of labor units is modelled by the above expression where h represents health (per capita expenditure on health) and s represents schooling (per capita expenditure on education).

Assuming that firms in the sector operate under a competitive market the profit function may be defined as follows:

$$\pi = TR - TC$$

(2.1.3)

$$TC = wL + rK + \Gamma$$

(2.1.4)

with: - w : wage rate;

- r : user cost of capital (the proxy often used is the interest rate);

- Γ : entry cost that is independent of other variables (inputs and output).

[6] This specification follows Weil (2001) and Bloom (2005).

Let's assume two output prices: the expected price (P_Q^e); and the current price (P_Q).

At the beginning of period *t*, firms base all their production on the expected price. However, should the actual price be set, firms follow an adjustment process. Since the producer moves according to expectations, mainly considering price, the producer's problem might be stated as follows:

Max: $\pi = P_Q^e Q - w L - r K - \Gamma$

(2.1.5)

Constraint: $Q = A_N (z L)^{\alpha} K^{\beta}$

(2.1.6)

Using the first order condition, the following optimal solutions are obtained, before price adjustment:

$$K^* = \left[\frac{\beta A P_Q^e}{r} \left(\frac{\alpha r}{\beta w} \right)^{\alpha} \right]^{\frac{1}{1-\alpha-\beta}}$$

(2.1.7)

$$z L^* = \frac{\alpha}{\beta} \cdot \frac{r}{w} \left[\frac{\beta A . P_Q^e}{r} \left(\frac{\alpha . r}{\beta . w} \right)^{\alpha} \right]^{\frac{1}{1-\alpha-\beta}}$$

(2.1.8)

$$L^* = \frac{\alpha}{\beta} \cdot \frac{r}{w} \left[\frac{\beta A . P_Q^e}{r} \left(\frac{\alpha . r}{\beta . w} \right)^{\alpha} \right]^{\frac{1}{1-\alpha-\beta}} . z^{-1}$$

(2.1.9)

After the price adjustment mechanism, the optimal solutions will be the following:

$$K^* = \left[\frac{\beta A P_Q^e}{r} \left(\frac{\alpha r}{\beta w} \right)^{\alpha} \right]^{\frac{1}{1-\alpha-\beta}} . \left[\frac{P_Q}{P_Q^e} \right]^{\phi_K}$$

(2.1.10)

$$L^* = \frac{\alpha}{\beta}.\frac{r}{w}\left[\frac{\beta.A.P_Q^e}{r}\left(\frac{\alpha r}{\beta w}\right)^{\alpha}\right]^{\frac{1}{1-\alpha-\beta}}\left[\frac{P_Q}{P_Q^e}\right]^{\phi_L}.z^{-1}$$

(2.1.11)

$$Q = A^{\frac{1}{1-\alpha-\beta}}.\alpha^{\frac{\alpha}{1-\alpha-\beta}}.\beta^{\frac{\beta}{1-\alpha-\beta}}.(P_{Q_{it}}^e)^{\frac{\alpha+\beta}{1-\alpha-\beta}}.w^{\frac{-\alpha}{1-\alpha-\beta}}.r^{\frac{-\beta}{1-\alpha-\beta}}.\left(\frac{P_Q}{P_Q^e}\right)^{\alpha\phi_L+\beta\phi_K}.z^{-\alpha}$$

(2.1.12)

The entry cost (much more of an administrative cost) is assumed to be independent of both inputs and output. In the optimization process it is treated as a constant variable and therefore does not appear on the optimizing *K* and *L*. However it directly affects the number of firms operating in the sector.

II.2. The Sales Supply equation

$$S_S = A^{\frac{1}{1-\alpha-\beta}}.\alpha^{\frac{\alpha}{1-\alpha-\beta}}.\beta^{\frac{\beta}{1-\alpha-\beta}}.\Im N(\Gamma).w^{\frac{-\alpha}{1-\alpha-\beta}}.r^{\frac{-\beta}{1-\alpha-\beta}}.P^{1+\alpha\phi_L+\beta}.\left(P_Q^e\right)^{-\alpha\phi_L-\beta\phi_K+\frac{\alpha+\beta}{1-\alpha-\beta}}.z^{-\alpha}$$

(2.2.1)

$$\frac{\partial N}{\partial \Gamma} \prec 0$$

The sales supply equation has been developed from the basic definition of sales:

$$S_S = (\Im N)\ .P_Q\ .q \qquad (2.2.2)$$

where $\Im = \sum_{j=1}^{N} \hbar_j$

and N being the total number of firms operating in the sector.

$'\hbar_j'$ is the $'j^{th}'$ firm's share in the sector sales activities (size characteristics) and $'q'$ is the individual firm's production. Whenever $\hbar_1 = \hbar_2 = ... = \hbar_N$, it simply means that all firms are identical and have the exact same shares of the sector's sales activities. In fact, the sum of all $'\hbar_j'$ will always be equal to one ($\Im = \sum_{j=1}^{N} \hbar_j = 1$). That's the reason why $'\hbar_j'$ does not appear in the sector's sales supply equation. However, we find it very important to highlight the fact that it is most likely that firms are not identical in the sector. If we consider '*Q*' as the total sector's production then the sales supply equation will be written as follows: $S_S = P_Q.Q$.

Importantly, it has to be highlighted that the expected price is nonlinear and has the following specification:

$$P_Q^e = \prod_{l=1}^{T} P_l^{\sigma_k}$$

(2.2.3)

with T being the total number of variables.

The sale equation can be expressed in growth terms by logging both sides of equation 2.2.1. and differentiating it with respect to time:

$$\frac{\dot{S}_S}{S_S} = \theta_1 \frac{\dot{A}}{A} + \left(\frac{\dot{\Im}}{\Im} + \frac{N(\dot{\Gamma})}{N(\Gamma)} \right) + \theta_2 \frac{\dot{P}}{P} + \theta_3 \frac{\dot{w}}{w} + \theta_4 \frac{\dot{r}}{r} + \sum_{l=1}^{T} \sigma_l \frac{\dot{P}_l}{P_l} + \theta_5 \frac{\dot{z}}{z}$$

(2.2.4)

where: - $\theta_1 = \dfrac{1}{1-\alpha-\beta}$

- $\theta_2 = 1 + \alpha\phi_L + \beta$
- $\theta_3 = \dfrac{-\alpha}{1-\alpha-\beta}$
- $\theta_4 = \dfrac{-\beta}{1-\alpha-\beta}$
- $\theta_5 = -\alpha$
- $\theta_6 = -\nu$

II.3. The sales demand equation

The sales demand equation can be formulated as follows:

$$S_D = (\Re D) P_Q . q \qquad (2.3.1)$$

where $\Re = \sum_{k=1}^{D} \upsilon_k$

with D being the total number of demanders of the sector's products and υ_k represents the k^{th} demander's size (share) of the sector's products demand. The demanders include: (1) firms; (2) private households; (3) government; as well as (4) foreign entities. The some of all $'\upsilon_s'$ will always be equal to one: $\Re = \sum_{k=1}^{D} \upsilon_k = 1$. Whenever all demanders are assumed to be identical, all $'\upsilon_s'$ will be equal. That is very less likely to happen though. Otherwise, with '*Q*' being the total demand, the sales equation can be written as follows:

$$S_D = P_Q.Q \tag{2.3.2}$$

Another way to present the sales demand function providing more details on explanatory variables will be to make use of the following equation:

$$S_D = P.\left[C_S (P_Q^e)^{\lambda_1}.(Y_d)^{\lambda_2}.(\Re D)^{\lambda_3} \prod_{j=1}^{m} X_j^{\chi_j}.\left(\frac{P_Q^e}{P_Q}\right)^{\Delta} \right] \tag{2.3.3}$$

Logging both sides and applying derivatives with respect to time the following growth equation is obtained:

$$\frac{\dot{S}_D}{S_D} = (1-\Delta)\frac{\dot{P}_Q}{P_Q} + (\lambda_1 + \Delta)\frac{\dot{P}_Q^e}{P_Q} + \lambda_2 \frac{(\dot{Y}_d)}{(Y_d)} + \lambda_3 \frac{\dot{\Re D}}{\Re D} + \chi_{j1}\frac{\dot{WY}}{WY} \tag{2.3.4}$$

where:

- Y_d : Gross National Disposable Income
- S_D : Sales Demand;
- X : Other variables affecting sales demand;
- WY : World Income.

II.4. Factor market

II.4.1. Labor

II.4.1.1. Labor Supply Equation

Under the assumption of perfect competition with *m* maximizing firms in the sector, we use a Cobb-Douglas function for labor. The aggregate labor supply is therefore presented as follows:

$$zL = C_L\left(\frac{w}{P_Q}\right)^{\psi_1}\left(\frac{S_S}{P_Q}\right)^{\psi_2}\left(\frac{P_Q}{P_Q^e}\right)^{\psi_3}(\vartheta(\rho D))^{\psi_4} \tag{2.4.1}$$

where: $\vartheta = \sum_{k=1}^{H} \vartheta_k$

It was specified earlier that ‘*D*’ stands for: the total number of demanders of market products. It includes: (1) households; (2) firms; (3) government; and (4) the rest of the world. When it comes to the economic units that supply for labor, it is correct to think of households (locals or even internationals). Households constitute only a portion 'ρ' of ‘*D*’ ($H = \rho D$). 'ϑ_k' is an index capturing the share of household ‘*k*’ in supplying effective labor (*zL*) for the given sector. 'ϑ_k' constitutes the link between the HPM (Household Product Model) and the factor market. When the household production activities are well functioning, the household can supply highly efficient and productive labor force. Household Production Models also appear on the demand side of the model through the sales demand. Household with higher production activities will demand more of the sales.

$$\frac{\dot{(zL)}}{(zL)} = \psi_1\left(\frac{\dot{w}}{w} - \frac{\dot{P}}{P}\right) + \psi_2\left(\frac{\dot{S}_S}{S_S} - \frac{\dot{P}}{P}\right) + \psi_3\left(\frac{\dot{P}_Q}{P_Q} - \frac{\dot{P}_Q^e}{P_Q^e}\right) + \psi_4\left(\frac{\dot{\vartheta}}{\vartheta} + \frac{\dot{\rho D}}{\rho D}\right) \tag{2.4.2}$$

II.4.1.2. Labor Demand Equation (Efficient Labor)

The demand for efficient labor is determined as follows:

$$zL = \alpha.\frac{S_S}{w}.\left(\frac{P_Q^e}{P_Q}\right)^{1+\beta\phi_K+(\alpha-1)\phi_L} \tag{2.4.3}$$

$$\frac{\dot{(zL)}}{(zL)} = \frac{\dot{S}_S}{S_S} - \frac{\dot{w}}{w} - (1+\beta\phi_K + (\alpha-1)\phi_L)\frac{\dot{P}_Q}{P_Q} + (1+\beta\phi_K + (\alpha-1)\phi_L)\frac{\dot{P}_Q^e}{P_Q^e} \tag{2.4.4}$$

$$\frac{\dot{(zL)}}{(zL)} = \frac{\dot{S}_S}{S_S} - \frac{\dot{w}}{w} + (1+\beta\phi_K + (\alpha-1)\phi_L)\left[\frac{\dot{P}_Q^e}{P_Q^e} - \frac{\dot{P}_Q}{P_Q}\right] \tag{2.4.5}$$

The model has implicitly included the level of unionisation that plays a major role in the labor market through wage determination. This provides more flexibility to this model and allows applying more sensitive policy simulation.

II.4.2. Capital

Concerning the market for capital, this study maintains the use of Cobb-Douglas function for maximizing firms.

II.4.2.1. Capital Supply Equation

$$K = C_K \left(\frac{r}{P_Q} \right)^{\gamma_1} \left(\frac{S_S}{P_Q} \right)^{\gamma_2} \left(\frac{P_Q}{P_Q^e} \right)^{\gamma_3} (\delta D)^{\gamma_4} \qquad (2.4.6)$$

$$\delta = \sum_{k=1}^{D} \delta_k$$

where δ_k is the k^{th} demander's share in capital supply.

$$\frac{\dot{K}}{K} = \gamma_1 \left(\frac{\dot{r}}{r} - \frac{\dot{P}_Q}{\dot{P}_Q} \right) + \gamma_2 \left(\frac{\dot{S}_S}{\dot{S}_S} - \frac{\dot{P}_Q}{\dot{P}_Q} \right) + \gamma_3 \left(\frac{\dot{P}_Q}{P_Q} - \frac{\dot{P}_Q^e}{P_Q^e} \right) + \gamma_4 \left(\frac{\dot{\delta}}{\delta} + \frac{\dot{D}}{D} \right) \qquad (2.4.7)$$

II.4.2.2. Capital Demand Equation

$$K = \beta \frac{S_S}{r} \left(\frac{P_Q^e}{P_Q} \right)^{1+\alpha\phi_L + (\beta-1)\phi_K} \qquad (2.4.8)$$

$$\frac{\dot{K}}{K} = \frac{\dot{S}_S}{S_S} - \frac{\dot{r}}{r} - [1 + \alpha\phi_L + (\beta - 1)\phi_K] \left(\frac{\dot{P}_Q}{P_Q} \right) + [1 + \alpha\phi_L + (\beta - 1)\phi_K] \left(\frac{\dot{P}_Q^e}{P_Q^e} \right) \qquad (2.4.9)$$

$$\frac{\dot{K}}{K} = \frac{\dot{S}_S}{S_S} - \frac{\dot{r}}{r} + [1 + \alpha\phi_L + (\beta - 1)\phi_K] \left(\frac{\dot{P}_Q^e}{P_Q^e} - \frac{\dot{P}_Q}{P_Q} \right) \qquad (2.4.10)$$

II.5. The Money Market

II.5.1. Money Supply Equation

The money market follows presumably the traditional route including a constant at which money supply is fixed and two shifters: consumer price index (*P*); and interest rate (*r*).

$$M_S = C_{M_S} . P^{\pi_1} . r^{\pi_2} \qquad (2.5.1)$$

$$\frac{\dot{M}_S}{M_S} = \pi_1 \left(\frac{\dot{P}}{P} \right) + \pi_2 \left(\frac{\dot{r}}{r} \right) \qquad (2.5.2)$$

II.5.2. Money Demand Equation

The demand for money is a also a twice differentiable function of several variables including qualitative households number, qualitative firms number and other traditional series such as interest rate, etc.

$$M^d = C_{M^d}.(\Re D)^{\nabla_1}.(\Im N)^{\nabla_2}.\left(\frac{r}{P_Q^e}\right)^{\nabla_3}.\left(\frac{S_S}{P_Q^e}\right)^{\nabla_4}.\left(\frac{P_Q}{P_Q^e}\right)^{\nabla_5} \quad (2.5.3)$$

$$\frac{\dot{M}^d}{M^d} = \nabla_1\left(\frac{\dot{\Re}}{\Re}+\frac{\dot{D}}{D}\right)+\nabla_2\left(\frac{\dot{\Im}}{\Im}+\frac{\dot{N}}{N}\right)+\nabla_3\left(\frac{\dot{r}}{r}-\frac{\dot{P}_Q^e}{P_Q^e}\right)+\nabla_4\left(\frac{\dot{S}_S}{S_S}-\frac{\dot{P}_Q^e}{P_Q^e}\right)+\nabla_5\left(\frac{\dot{P}_Q}{P_Q}-\frac{\dot{P}_Q^e}{P_Q^e}\right)$$

(2.5.4)

II.6. Entry/Exit

The present model specification includes an entry/exit set of equations that constitute the cartilage (point of junction) between supply, demand, and factors market. In fact, firms do enter and exit the sector quite often and that should be reflected in the modelling exercise to make it more realistic. Firms are mainly attracted by any excess made by higher sales as compared to equilibrium profit. And firms will be likely to live once the sales go below the equilibrium profit.

$$\frac{\dot{N}}{N} = C_E(S_S - \pi^e) \quad (2.6.1)$$

N is the total number of firms operating in the sector. 'π^e'is the equilibrium profit level of individual firms in the sector. A thorough modelling of 'π^e' is required in order to have a perfect understanding of firms movements in the sector and this will be investigated in further studies. In fact, several factors are involved in the determination of'π^e'. In this research, equation 2.6.1 is described as an identity that is solved out through the all system of reduced form equations. This equation is not estimated.

III. A note on 'expected price P_Q^e'

It is plausible and rather realistic to make the assumption that consumers as well as producers take into consideration expected prices and adjust their consumption or production accordingly. 'P_Q^e' is assumed to be a nonlinear function. Its formulation depends on many variables. The literature argues that previous and actual prices are the determinants of people expectations[7]. Expected price is therefore defined as the mean (geometric mean) of $P_{Q(t-1)}$ (lagged actual price) and $P^{e}_{Q(t-1)}$ (lagged expected price)[8]. It can be formulated as follows:

$$\ln P^{e}_{Qt} = \varpi \ln P_{Q(t-1)} + (1-\varpi)\ln P^{e}_{Q(t-1)} \qquad \varpi \in (0,1) \tag{3.1}$$

Furthermore, this equation can be developed such as:

$$\ln P^{e}_{Qt} = \varpi \ln P_{Q(t-1)} + \varpi(1-\varpi)\ln P_{Q(t-2)} + \varpi(1-\varpi)^{2} \ln P_{Q(t-3)} \tag{3.2}$$

$$\ln P^{e}_{Q_t} = \ln\left[P^{\varpi}_{Q_{(t-1)}} . P^{\varpi(1-\varpi)}_{Q_{(t-2)}} . P^{\varpi(1-\varpi)^2}_{Q_{(t-3)}}\right] \tag{3.3}$$

As it was mentioned earlier, the expected price is a nonlinear function of actual prices and can be written in lag form:

$$P^{e}_{Q_t} = \prod_{j=1}^{n} P^{\sigma_j}_{Q_{(t-j)}} \tag{3.4}$$

where: $\sigma_j = \varpi(1-\varpi)^{j-1}$.

The general assumption that supports the theory of rational expectation stipulates that no information is wasted in the economic system. The use of an average weight for expectations, although it is not a completely truthful representation of the reality, has helped improving models accuracy as compared to models that do not make any reference to price expectations. Considering equation 3.1, in the event that ϖ equals one, it would mean that the product's expected price in period *t* ($P^{e}_{Q_t}$) is exactly equal to the product price in period *t-1* ($P_{Q_{(t-1)}}$). In other words, people expect that price remains unchanged. The smaller is the value of ϖ, the larger is the gap between $P^{e}_{Q_t}$ and $P_{Q_{(t-1)}}$.

The other plausible assumption to consider may be that people make reference to domestic inflation rate while setting price expectations. Therefore the value of ϖ will be assimilated to the CPI (Consumer Price Index). However, to some extent, consumers might rather consider the trend of international indicators such as the oil price while fixing their price expectations. In this analysis, since all clusters of the South African economy are considered, it will be

[7] Muth, J.F. "Rational Expectations and the Theory of Price Movements", Econometrica, 29, 1961.
[8] Kim KH. "Empirical Evidence of Forecasting Improvements from Disaggregating the US Economy", Working Paper, University of Chicago, October 2006.

unrealistic to assume that the market agents, mainly the consumers, have a perfect understanding of the dynamic of international markets. Therefore the model estimates expected price as a function of the equilibrium price plus some error terms[9]. Expectations vary according to different variables such as: (1) the type of economic system; (2) the quantity of information available; (3) the volatility of leading indicators; (4) the level of market openness; etc.

The choice of an appropriate series for expected price matters. Any wrongdoing will most likely mislead the model by raising the number of biased estimates. As it was mentioned earlier, the basic and simplest approach used in order to determine price expectations consist of conducting a survey and ask directly respondents about their expectations on market prices. The University of Michigan has made use of this approach for several years. Other alternative approaches of determining price expectations include: (1) extrapolative expectation (Goodwin 1947); (2) adaptive expectation (Nerlove 1958); (3) classical theory of expectation (Schultz et al. 1958); (4) the conditional (weighted) expectation theory (Muth 1961); (5) the C-P method (Carlson and Parkin, 1975); (6) rational expectation (Lucas 1981); (7) the inflation indexed bonds (Kitamura, 1997); (8) the expectation-augmented Philips curve (Hori et al. 2003); etc.

Since theories of expectations have yet not been able to predict human behaviour, the best they can do is to establish links between predictions and market equilibrium dynamics. The real measure of human behaviour in the determination of expectations remains unclear.
In reference to the general understanding of price expectations, equation 3.1 can be reformulated by deriving it with respect to time as follows:

$$\frac{\dot{P}^{e}_{Q_t}}{P^{e}_{Q_t}} = \varpi \frac{\dot{P}_{Q_{t-1}}}{P_{Q_{t-1}}} + \varpi (1 - \varpi) \frac{\dot{P}^{e}_{Q_{t-1}}}{P^{e}_{Q_{t-1}}} + \varepsilon \qquad (3.5)$$

IV. Including leading indicators

Following the supportive arguments that was raised earlier for the use of reduced form equations (RFE), the use of leading indicators such as Money (M2) and Stock Prices (SP) enhances the model's forecasting ability. It is interesting to bring in a comparative analysis

[9] The error term here represents all unobserved elements that affect the determination of price expectations.

when it comes to forecasting ability between the ARLI (3) approach and the RFE. Once the two leading indicators are included with allowing three lag terms, the reduced-form equations can be written as follows:

$$\ln\left(\frac{S_{St}}{S_{S(t-1)}}\right) \approx \theta_0^{''} + \theta_1^{''} S_{S(t-1)} + \theta_2^{''} \ln\left(\frac{P_{Q_{(t-1)}}}{P_{Q_{(t-2)}}}\right) + \theta_3^{''} \ln\left(\frac{P_{Q_{(t-2)}}}{P_{Q_{(t-3)}}}\right) + \theta_4^{''} \ln\left(\frac{P_{Q_{(t-3)}}}{P_{Q_{(t-4)}}}\right) + \theta_5^{''} \ln\left(\frac{M_{(t-1)}}{M_{(t-2)}}\right)$$

$$+ \theta_6^{''} \ln\left(\frac{SP_{Q_{(t-1)}}}{SP_{Q_{(t-2)}}}\right) + \theta_7^{''} \ln\left(\frac{Y_t}{Y_{(t-1)}}\right) + \theta_8^{''} \ln\left(\frac{\dot{A}_t}{A_{(t-1)}}\right) + \varepsilon_{St} \qquad (4.1)$$

The subscript t represents the time period considered.

$$\ln\left(\frac{P_{Q_t}}{P_{Q_{(t-1)}}}\right) \approx \sigma_0^{''} + \sigma_1^{''} S_{S(t-1)} + \sigma_2^{''} \ln\left(\frac{P_{Q_{(t-1)}}}{P_{Q_{(t-2)}}}\right) + \sigma_3^{''} \ln\left(\frac{P_{Q_{(t-2)}}}{P_{Q_{(t-3)}}}\right) + \sigma_4^{''} \ln\left(\frac{P_{Q_{(t-3)}}}{P_{Q_{(t-4)}}}\right) + \sigma_5^{''} \ln\left(\frac{M_{(t-1)}}{M_{(t-2)}}\right)$$

$$+ \sigma_6^{''} \ln\left(\frac{SP_{Q_{(t-1)}}}{SP_{Q_{(t-2)}}}\right) + \sigma_7^{''} \ln\left(\frac{Y_t}{Y_{(t-1)}}\right) + \sigma_8^{''} \ln\left(\frac{A_t}{A_{(t-1)}}\right) + \varepsilon_{Pt} \qquad (4.2)$$

The RFE include both leading indicators and lag terms. This type of specification contains some similarities with the ARLI (3) models however the literature has provided enough evidence that the RFE forecast much better (see Zellner et al. 2005). It is of good interest to highlight the reader on how this is applicable for South African data.

The simple ARLI (3) models would be formulated as follows:

$$\ln\left(\frac{S_{St}}{S_{S(t-1)}}\right) \approx \theta_0^{'''} + \theta_1^{'''} S_{S(t-1)} + \theta_2^{'''} S_{S(t-2)} + \theta_3^{''} S_{S(t-3)} + \theta_4^{''} \ln\left(\frac{SP_{Q_{(t-3)}}}{SP_{Q_{(t-4)}}}\right) + \theta_5^{''} \ln\left(\frac{M_{(t-1)}}{M_{(t-2)}}\right) + \varepsilon_{St}^{'}$$

(4.3)

V. The Reduced Form Equations Disaggregated by sectors (RFE-DA)

As it was mentioned earlier, the set of disaggregated reduced form equations (RFE-DA) are an expansion of the aggregate reduced form equations (RFE) including all industrial sectors[10] (see appendix 1). The RFE-DA can therefore be formulated as follows:

[10] The subscript *i* represents the development sector. The model investigates 10 development sectors of the South African economy.

Sector's output (sales supply):

$$\ln\left(\frac{S_{it}}{S_{i(t-1)}}\right) \approx \lambda_0 + \lambda_1 S_{S,i(t-1)} + \lambda_2 \ln\left(\frac{P_{Q_{i(t-1)}}}{P_{Q_{i(t-2)}}}\right) + \lambda_3 \ln\left(\frac{P_{Q_{i(t-2)}}}{P_{Q_{i(t-3)}}}\right) + \lambda_4 \ln\left(\frac{P_{Q_{i(t-3)}}}{P_{Q_{i(t-4)}}}\right) + \lambda_5 \ln\left(\frac{M_{(t-1)}}{M_{(t-2)}}\right)$$

$$+ \lambda_6 \ln\left(\frac{SP_{Q_{(t-1)}}}{SP_{Q_{(t-2)}}}\right) + \lambda_7 \ln\left(\frac{Y_{d,t}}{Y_{d,(t-1)}}\right) + \lambda_8 \ln\left(\frac{r_t}{r_{(t-1)}}\right) + \lambda_9 \ln\left(\frac{w_{it}}{w_{i(t-1)}}\right) + \lambda_{10} \ln\left(\frac{H_t}{H_{(t-1)}}\right)$$

$$+ \lambda_{11} \ln\left(\frac{z_{it}}{z_{i(t-1)}}\right) + \lambda_{10} \ln\left(\frac{\vartheta_t}{\vartheta_{(t-1)}}\right) + \lambda_{12} \ln\left(\frac{\Gamma_{it}}{\Gamma_{i(t-1)}}\right) + \lambda_{13} \ln\left(\frac{WY_t}{WY_{(t-1)}}\right) + \lambda_{14} \ln\left(\frac{A_{it}}{A_{i(t-1)}}\right) + \upsilon_{St}$$

(5.1)

Sector's output price:

$$\ln\left(\frac{P_{Q_{it}}}{P_{Q_{i(t-1)}}}\right) \approx \iota_0 + \iota_1 S_{S,i(t-1)} + \iota_2 \ln\left(\frac{P_{Q_{i(t-1)}}}{P_{Q_{i(t-2)}}}\right) + \iota_3 \ln\left(\frac{P_{Q_{i(t-2)}}}{P_{Q_{i(t-3)}}}\right) + \iota_4 \ln\left(\frac{P_{Q_{i(t-3)}}}{P_{Q_{i(t-4)}}}\right) + \iota_5 \ln\left(\frac{M_{(t-1)}}{M_{(t-2)}}\right)$$

$$+ \iota_6 \ln\left(\frac{SP_{Q_{(t-1)}}}{SP_{Q_{(t-2)}}}\right) + \iota_7 \ln\left(\frac{Y_{d,t}}{Y_{d,(t-1)}}\right) + \iota_8 \ln\left(\frac{r_t}{r_{(t-1)}}\right) + \iota_9 \ln\left(\frac{w_{it}}{w_{i(t-1)}}\right) + \iota_{10} \ln\left(\frac{H_t}{H_{(t-1)}}\right) + \iota_{10} \ln\left(\frac{\vartheta_t}{\vartheta_{(t-1)}}\right)$$

$$+ \iota_{11} \ln\left(\frac{Op_t}{Op_{(t-1)}}\right) + \iota_{12} \ln\left(\frac{\Gamma_{it}}{\Gamma_{i(t-1)}}\right) + \iota_{13}\left(\frac{WY_t}{WY_{(t-1)}}\right) + \iota_{14} \ln\left(\frac{A_{it}}{A_{i(t-1)}}\right) + \upsilon_{PS}$$

(5.2)

This model specification can be used to forecast Sales and Inflation rates at sectoral level. The RFE-DA follow a continuous time specification. Applying log approximation, these can be transformed into discrete model specification.

$$\frac{\dot{S}_{S,it}}{S_{S,it}} = \frac{d \ln S_{S,it}}{dt} \approx \ln\left(\frac{S_{S,it}}{S_{S,i(t-1)}}\right) \qquad (5.3)$$

VI. Transfer Function

Transfer function equations are obtained using a matrix of lag operators with derivations of selected transfer functions (see Kim, 2006). Referring to equation 5.1 the following equations can be obtained for: the sales supply; the sales demand; and the entry/exit equations.

$$\ln\left(\frac{S_{Si,t}}{S_{Si,t-1}}\right) = \ln\left(\frac{N_{i,t}}{N_{i,t-1}}\right) + \kappa_{1,i}\ln\left(\frac{w_{i,t}}{w_{i,t-1}}\right) + \kappa_{2,i}\ln\left(\frac{r_t}{r_{t-1}}\right) + \kappa_{3,i}\ln\left(\frac{A_{i,t}}{A_{i,t-1}}\right) + \kappa_{4,i}\ln\left(\frac{z_{i,t}}{z_{i,t-1}}\right)$$

$$+ \kappa_{5,i}\ln\left(\frac{\Gamma_{i,t}}{\Gamma_{i,t-1}}\right) + \lambda(L)\ln\left(\frac{P_{i,t}}{P_{i,t-1}}\right) + \varepsilon_{Ti,t} \quad (6.1)$$

$$\ln\left(\frac{S_{Di,t}}{S_{Di,t-1}}\right) = \Delta_{1,i}\ln\left(\frac{Y_{d,t}}{Y_{d,t-1}}\right) + \Delta_{2,i}\ln\left(\frac{WY_t}{WY_{t-1}}\right) + \Delta_{3,i}\ln\left(\frac{D_{i,t}}{D_{i,t-1}}\right) + \gamma(L)\ln\left(\frac{P_{Qi,t}}{P_{Qi,t-1}}\right) + \mu_{Ti,t}$$

(6.2)

$$\ln\left(\frac{N_{i,t}}{N_{i,t-1}}\right) = \delta_{0,i} + \delta_{1,i}S_{i,t-1} + v_{Ti,t} \quad (6.3)$$

where:

- $\lambda(L)$ and $\gamma(L)$: lag operators
- X : set of other exogenous variables obtained from the ARLI (3) model: *SP* (Stock Prices) and M (Money Supply: M2);
- $\ln\left(\frac{S_{i,t}}{S_{i,t-1}}\right) = s_{i,t}$;
- $\ln\left(\frac{N_{i,t}}{N_{i,t-1}}\right) = n_{i,t}$;
- $\ln\left(\frac{w_{i,t}}{w_{i,t-1}}\right) = n_{i,t}$;
- $\ln\left(\frac{r_{i,t}}{r_{i,t-1}}\right) = n_{i,t}$;
- $\ln\left(\frac{A_{i,t}}{A_{i,t-1}}\right) = n_{i,t}$;
- $\ln\left(\frac{z_{i,t}}{z_{i,t-1}}\right) = z_{i,t}$;
- $\ln\left(\frac{\Gamma_{i,t}}{\Gamma_{i,t-1}}\right) = \Gamma_{i,t}$;

The structural equations model can be presented under matrix form:

$$\begin{bmatrix} 1 & -\lambda(L) & -1 \\ 1 & -\gamma(L) & 0 \\ 0 & 0 & 1 \end{bmatrix} . \begin{bmatrix} s_{i,t} \\ p_{i,t} \\ n_{i,t} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \delta_{0,i} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \delta_{1,i} \end{bmatrix} S_{i,t-1} + \begin{bmatrix} \kappa_{1,i} \\ 0 \\ 0 \end{bmatrix} w_{i,t} + \begin{bmatrix} \kappa_{2,i} \\ 0 \\ 0 \end{bmatrix} r_t + \begin{bmatrix} \kappa_{3,i} \\ 0 \\ 0 \end{bmatrix} a_{i,t} + \begin{bmatrix} \kappa_{4,i} \\ 0 \\ 0 \end{bmatrix} z_{i,t}$$

$$+ \begin{bmatrix} \kappa_{5,i} \\ 0 \\ 0 \end{bmatrix} \Gamma_{i,t} + \begin{bmatrix} \kappa_{6,i} \\ 0 \\ 0 \end{bmatrix} X_t + \begin{bmatrix} 0 \\ \Delta_{1,i} \\ 0 \end{bmatrix} y_t + \begin{bmatrix} 0 \\ \Delta_{2,i} \\ 0 \end{bmatrix} wy_t + \begin{bmatrix} 0 \\ \Delta_{3,i} \\ 0 \end{bmatrix} d_t + \begin{bmatrix} \varepsilon_{Ti,t} \\ \mu_{Ti,t} \\ v_{Ti,t} \end{bmatrix} \tag{6.4}$$

In order to obtain the final equations (MMM-DA), it is important to multiply both side of equation 6.4 by the matrix A^* ($A^* = \det A.A^{-1}$), with:

$$A = \begin{bmatrix} 1 & -\lambda(L) & -1 \\ 1 & -\gamma(L) & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{and} \quad A^{-1} = \frac{1}{\det A} \begin{bmatrix} -\gamma(L) & \lambda(L) & -\gamma(L) \\ -1 & 1 & -1 \\ 0 & 0 & \lambda(L)-\gamma(L) \end{bmatrix}$$

Therefore: $A^* = \begin{bmatrix} -\gamma(L) & \lambda(L) & -\gamma(L) \\ -1 & 1 & -1 \\ 0 & 0 & \lambda(L)-\gamma(L) \end{bmatrix}$

After multiplying both sides of equation 6.4 by A^* :

$$[\lambda(L)-\gamma L)].\begin{bmatrix} s_{i,t} \\ p_{i,t} \\ n_{i,t} \end{bmatrix} = \begin{bmatrix} -\gamma(L)\delta_{0,i} \\ -\delta_{0,i} \\ \delta_{0,i}[\lambda(L)-\gamma(L)] \end{bmatrix} + \begin{bmatrix} -\gamma(L)\delta_{1,i} \\ -\delta_{1,i} \\ \delta_{1,i}[\lambda(L)-\gamma(L)] \end{bmatrix} S_{i,t-1} + \begin{bmatrix} -\gamma(L)\kappa_{1,i} \\ -\kappa_{1,i} \\ 0 \end{bmatrix} w_{i,t}$$

$$+ \begin{bmatrix} -\gamma(L)\kappa_{2,i} \\ -\kappa_{2,i} \\ 0 \end{bmatrix} r_t + \begin{bmatrix} -\gamma(L)\kappa_{3,i} \\ -\kappa_{3,i} \\ 0 \end{bmatrix} a_{i,t} + \begin{bmatrix} -\gamma(L)\kappa_{4,i} \\ -\kappa_{4,i} \\ 0 \end{bmatrix} z_{i,t} + \begin{bmatrix} -\gamma(L)\kappa_{5,i} \\ -\kappa_{5,i} \\ 0 \end{bmatrix} \Gamma_{i,t}$$

$$+ \begin{bmatrix} -\gamma(L)\kappa_{6,i} \\ -\kappa_{6,i} \\ 0 \end{bmatrix} X_t + \begin{bmatrix} \lambda(L)\Delta_{1,i} \\ \Delta_{1,i} \\ 0 \end{bmatrix} y_t + \begin{bmatrix} \lambda(L)\Delta_{2,i} \\ \Delta_{2,i} \\ 0 \end{bmatrix} wy_t + \begin{bmatrix} \lambda(L)\Delta_{3,i} \\ \Delta_{3,i} \\ 0 \end{bmatrix} d_t +$$

$$+ \begin{bmatrix} -\gamma(L)\varepsilon_{Ti,t} + \lambda(L)\mu_{Ti,t} - \gamma(L)v_{Ti,t} \\ -\varepsilon_{Ti,t} + \mu_{Ti,t} - v_{Ti,t} \\ [\lambda(L)-\gamma(L)]v_{Ti,t} \end{bmatrix}$$

(6.5)

Equation 6.5 can be transformed into a system of linear equations for both price and sales supply:

$$[\lambda(L)-\gamma(L)].s_{i,t} = -\gamma(L)\delta_{0,i} - \gamma(L)\delta_{1,i}S_{i,t-1} - \gamma(L)\kappa_{1,i}w_{i,t} - \gamma(L)\kappa_{2,i}r_t - \gamma(L)\kappa_{3,i}a_{i,t} - \gamma(L)\kappa_{4,i}z_{i,t}$$

$$-\gamma(L)\kappa_{5,i}\Gamma_{i,t} - \gamma(L)\kappa_{6,i}X_t + \lambda(L)\Delta_{1,i}y_t + \lambda(L)\Delta_{2,i}wy_t + \lambda(L)\Delta_{3,i}d_t - \gamma(L)\varepsilon_{Ti,t}$$
$$+\lambda(L)\mu_{Ti,t} - \gamma(L)v_{Ti,t}$$

(6.6)

$$[\lambda(L)-\gamma(L)].P_{i,t} = -\delta_{0,i} - \delta_{1,i}S_{i,t-1} - \kappa_{1,i}w_{i,t} - \kappa_{2,i}r_t - \kappa_{3,i}a_{i,t} - \kappa_{4,i}z_{i,t} - \kappa_{5,i}\Gamma_{i,t} - \kappa_{6,i}X_t + \Delta_{1,i}y_t$$

$$+\Delta_{2,i}wy_t + \Delta_{3,i}d_t - \varepsilon_{Ti,t} + \mu_{Ti,t} - v_{Ti,t}$$

(6.7)

VII. The MMM-DA (Transfer equations)

a) Sector's supply equation:

$$\ln\left(\frac{S_{S,it}}{S_{S,i(t-1)}}\right) \approx \int_{0i} + \int_{1i} S_{S,i(t-1)} + \int_{2i} S_{S,i(t-2)} + \int_{3i} S_{S,i(t-3)} + \int_{4i} \ln\left(\frac{A_{it}}{A_{i(t-1)}}\right) + \int_{5i} \ln\left(\frac{M_{(t-1)}}{M_{(t-2)}}\right)$$

$$+ \int_{6i} \ln\left(\frac{SP_{(t-1)}}{SP_{(t-2)}}\right) + \int_{7i} \ln\left(\frac{Y_{dt}}{Y_{d(t-1)}}\right) + \int_{8i} \ln\left(\frac{r_t}{r_{(t-1)}}\right) + \int_{9i} \ln\left(\frac{w_{it}}{w_{i(t-1)}}\right) + \int_{10i} \left(\frac{\dot{\Re}}{\Re} + \frac{\dot{D}_t}{D_{(t-1)}}\right)$$

$$+ \int_{11i} \ln\left(\frac{z_{it}}{z_{i(t-1)}}\right) + \int_{12i} \ln\left(\frac{\Gamma_{it}}{\Gamma_{i(t-1)}}\right) + \int_{13i} \ln\left(\frac{WY_t}{WY_{(t-1)}}\right) + \varepsilon_{Sit}$$

(7.1)

b) Sector's price equation:

$$\ln\left(\frac{P_{Q,it}}{P_{Q,i(t-1)}}\right) \approx \kappa_{0i} + \kappa_{1i} S_{S,i(t-1)} + \kappa_{2i} S_{S,i(t-2)} + \kappa_{3i} S_{S,i(t-3)} + \kappa_{4i} \ln\left(\frac{\dot{A}_{it}}{A_{i(t-1)}}\right) + \kappa_{5i} \ln\left(\frac{M_{(t-1)}}{M_{(t-2)}}\right)$$

$$+ \kappa_{6i} \ln\left(\frac{SP_{(t-1)}}{SP_{(t-2)}}\right) + \kappa_{7i} \ln\left(\frac{Y_{dt}}{Y_{d(t-1)}}\right) + \kappa_{8i} \ln\left(\frac{r_t}{r_{(t-1)}}\right) + \kappa_{9i} \ln\left(\frac{w_{it}}{w_{i(t-1)}}\right) + \kappa_{10i} \left(\frac{\dot{\Re}}{\Re} + \frac{\dot{D}_t}{D_{(t-1)}}\right)$$

$$+ \kappa_{11i} \ln\left(\frac{z_{it}}{z_{i(t-1)}}\right) + \kappa_{12i} \ln\left(\frac{\Gamma_{it}}{\Gamma_{i(t-1)}}\right) + \kappa_{13i} \ln\left(\frac{WY_t}{WY_{(t-1)}}\right) + \varepsilon_{Pit}$$

(7.2)

VIII. The Iterative SUR (Seemingly Unrelated Regressions): Bayesian versus Non-Bayesian Perspective

In order to estimate the MMM-DA model, the ISUR have been utilized in this research. The ISUR method provides estimates using GLS (Generalised Least Square) specification. Among several other advantages, the use of ISUR allows for correction of contemporaneous correlation and heteroskedasticity biases related to the different cross-sections. The iterative SUR approach used in this paper permits to estimate transfer equations with a different coefficient vectors[11]. The correlation of cross sections disturbances increases efficiency and that is observable using the ISUR approach.

One could suggest the use of a rather Bayesian approach such as MCMC (Markov Chain Monte Carlo Simulations), as more described at a later stage, in comparison to the GLS.

[11] When a restriction is imposed to have the same coefficient vectors across sectors, we call it complete shrinkage. The use complete shrinkage will be discussed later on in order to provide improvement to our estimates.

Indeed, the paper compares the different approaches considering criteria of good forecast (i.e. RMSE and MAE). However, the use of MCMC require a good specification of the likelihood function while the iterative SUR method provides the MLE (Maximum Likelihood Estimators) using an algorithm. Although, MCMC is based on the use of an algorithm for computing the posterior distribution, it requires a prior and a likelihood function. While iterating the SUR with a normal likelihood function with a flat prior, the modal value of the distribution is obtained. And the modal value will be optimal to a zero-one loss function. In large sample, the posterior distribution is normal and the posterior-mean equals the maximum likelihood estimator (MLE). The posterior shapes up to be normal and the '*t-statistic*' is approximated based on the normal distribution. The iterative SUR procedure iterates in the MLE under a broad range of condition. In the case of large sample situations, the MLE will be equal to the mean of the posterior and to the modal value. And that is a Bayesian estimate.

IX. Improving predictions using Shrinkage Techniques

Stein shrinkage techniques constitute an appropriate way to improve predictions of the MMM-DA. The use of shrinkage techniques has produced better outcomes on country as well as regional forecasting. The Stein's Mean approach consists of estimating the vector mean using a quadratic loss function including the goodness of fit. The model specifications of the balanced loss function as well as the squared error loss function (Dey *et al*, 1994) are as follows:

- Balanced loss function:

$$L(\hat{\theta},\theta) = w(y-\hat{\theta})'(y-\hat{\theta}) + (1-w)(\hat{\theta}-\theta)'(\hat{\theta}-\theta) \qquad (8.1)$$

- Squared error loss function:

$$L_b = w(\iota' y - \hat{T})^2 + (1-w)(\hat{T}-T)^2 \qquad (8.2)$$

With: - θ: the vector of means

- y: the series considered
- w: a weigh imposed
- T: total of the mean observation vectors

The shrinkage techniques make use of future vector of observations presented in quadratic loss functions (squared errors) to predict future total using the predictive means. For each sector, specific mean vectors need to be estimated and their total need to be computed. Considering the sectors' disparities, it is important to generate observation vectors using SUR (Seemingly Unrelated) models:

$$y_{ni} = x_{ni}\gamma_{ni} + \mu_{ni} \tag{8.3}$$

with $n = 1, \ldots\ldots\ldots\ldots, 10$

$i = 1, \ldots\ldots\ldots\ldots, 25$

The shrinkage estimation of the γ_{ni} will affect prediction of a future observation vector and the total. For the n_i mean, an informative prior needs to be added (Leonard *et al*, 2001) when considering both the prior and a normal likelihood. The shrinkage estimators are accepted under lemma conditions for quadratic loss functions. For disaggregated model it is important to consider sectoral shrinkage assumptions regarding the estimates of the vectors of means. The evaluation of the predictive performance of the shrinkage estimates requires running comparative analysis with existing forecasting models looking at criteria like; the MAEs; the RMSEs; and the Akaike or Schwarz criteria.

The shrinkage can be standard, as it has been described above, or complete. The complete shrinkage, much easier to perform, assumes common coefficient vectors for each cross-section. Similarly to the standard shrinkage, it allows reducing the variance and therefore reducing the RMSE (and MAE). Complete shrinkage entails very strong assumptions on the functioning of sectors and that will most likely not reflect the structural reality of the model.

X. Imposing restrictions

It is mostly recommended to estimate equations with fewer parameters. To this extent we have decided to reduce the number of parameter estimates referring to results obtained from the Wald test on parameters restrictions. In order to reduce the number of parameters in the model, a careful process (see Zellner and Chen, 2000) can be followed.

Considering our marshallian model with both endogenous and predetermined variables with L being the lag operator, we can write the variables in vector form:

Set of endogenous variables and exogenous variables (random) per sector:

$$\underbrace{\begin{pmatrix} y_{1t} \\ y_{2t} \\ y_{3t} \\ y_{4t} \\ y_{5t} \\ y_{6t} \\ y_{7t} \end{pmatrix}}_{Y'} = \begin{pmatrix} (1-L)\ln S_{St} \\ (1-L)\ln WY_t \\ (1-L)\ln w_t \\ (1-L)\ln r_t \\ (1-L)\ln Y_{dt} \\ (1-L)\ln \Gamma_t \\ (1-L)\ln(D)_t \end{pmatrix} \quad \text{with} \quad \underbrace{\begin{pmatrix} - \\ \alpha_{21} \\ \alpha_{31} \\ \alpha_{41} \\ \alpha_{51} \\ \alpha_{61} \\ \alpha_{71} \end{pmatrix}}_{\alpha' \text{(matrix of coefficient)}}$$

Vector x of lagged variables in reference to the ARLI (3) model:

$$x'_{1t} = \left[1; S_{S,t-1}; S_{S,t-2}; S_{S,t-3}; (1-L)\ln M2_{t-1}; (1-L)\ln SP_{t-1}\right] \tag{9.1}$$

The two matrices equalities can be written as follows:

$$y_{1t} = \alpha_{21}y_{2t} + \alpha_{31}y_{3t} + \alpha_{41}y_{4t} + \alpha_{51}y_{5t} + \alpha_{61}y_{6t} + \beta_1 x'_{1t} + \mu_{1t} \tag{9.2}$$

$$y_1 = \alpha_1 Y_1 + \beta_1 X_1 + \mu_1 \tag{9.3}$$

Assuming that π is the coefficient matrix of X, the unrestricted MMM-DA equations can be written as follows:

$$Y_1 = \prod_1 X + \varepsilon_1 \tag{9.4}$$

The Restricted MMM-DA (RMMM-DA) is obtained by replacing equation 9.4 into equation 9.3:

$$y_1 = \alpha_1 \prod_1 X + \beta_1 X_1 + \varepsilon_1 + \mu_1 \tag{9.5}$$

and

$$y_1 = \theta_1 \overline{M} + (\varepsilon_1 + \mu_1) \tag{9.6}$$

with: $\overline{M}$ being a full column rank such as $\overline{M} = (\prod_1 X; X_1)$ and $\varepsilon_1 + \mu_1 = \nu_1$

XI. MCMC in estimating the SUR model

As a highly appropriate method in micro-econometric applied researches, MCMC is a method that has helped improving the exploration of the posterior density using algorithms. Referring to the fact that the posterior distribution $\pi^*(\theta)$ is not normal, building simulations-based estimates in the given distribution is the core focus of the MCMC procedure[12]. The use of MCMC is more relevant in cases where θ is very large. In this study, for 10 economic sectors of the South African economy with as set of equations solved as reduced forms containing 12 to 13 variables per sectors, the space dimension to be explored rises to a certain maximum of more or less 840. That is relatively large and therefore the use of MCMC is justified.

The MCMC method consists of using the underpinning theory of 'Markov Chains' as often referred to by the parameter space using simulation methods developed under the theory of Monte Carlo. In other, world, the problem is respecified as a Markov chain with $\pi(\theta)$ being the equilibrium distribution of the chain. The estimates of the integrals are obtainable using Monte Carlo Simulations (see Rossi et al) as the simulation process consists of several draws (5000 iterations in our case) used to produce estimates of θ or any function of θ. Random variables of θ are generated using a sequential process until the iteration process is stopped at time T. Therefore, the move from θ_t to θ_{t+1} is operated using the principle of transition matrix inducing that θ_{t+1} is conditioned by θ_t. Assuming D_C, a conditional distribution, the use of Markov chains allows to create a joint distribution of the process. MCMC requires adamant restrictions to be specified for the posterior distribution to converge toward a unique distribution. Once the convergence condition is met by the posterior distribution that is already stationary, the Markov chain process used under Monte Carlo simulations can produce reliable estimates (Rossi et al). After a number of draws T, MCMC can be used to construct the posterior expectation of θ or a function of θ $f(\theta)$. Referring to its assumption on convergence, MCMC can validly be said to be asymptotic.

Let an ergodic chain:

$$\lim_{T\to\infty} \frac{1}{T} \sum f(\theta_t) = E_\pi[f(\theta)]$$

with $E_\pi[f(\theta)]$ being the expected value of the posterior distribution.

[12] Rossi P.E.; Allenby G.M.; and McCulloch R.: 'Bayesian Statistics and Marketing', Wiley Series in Probability and Statistics, 2005.

Most often, the use of Markov chains entails that the first set of draws is disregarded since it constitutes the initial observations. In fact, the draws start to be considered from the distribution once the initial period is passed and therefore the chain starts equilibrating.

$$E_{\pi}[f(\theta)] = \frac{1}{T-S}\sum_{t=S+1}^{T} f(\theta_t),$$

with S being the period of initial observations excluded from the draws ('burn in' period).

The various draws of θ might be depending from one another and that renders the process to be a non iid process. However, in most cases, in the literature on large number simulation experiments the draws would be assumed to be independent. MCMC requires very specific types of algorithms and the Gibbs Sampler is one of the most popular applications of MCMC.

In this paper, we make use of the Metropolis-Hastings algorithm to update the parameters. Assuming the set of parameters α (dimensional coefficient vector) and Ω (squared matrix sized up based on the number of cross-section, with diagonal elements being $\{\omega_1^2,...,\omega_m^2\}$ and off-diagonal elements being ω_{ij}), the process can be summarized as follows (Zellner et al, 2008): (1) initialization of the parameters α and Ω as the maximum likelihood estimates; (2) sample both coefficient parameters; and (3) repeat the sampling process throughout several iterations. The end result of such a process is to obtain a sample from the posterior density function after the 'burn in' period.

From the parameter α, we generate a candidate parameter $\alpha^{(t)}$ at the t$^{\text{th}}$ iteration with a maximum likelihood estimate Ω_{MLE}. The probability to accept the candidate draw is ρ_α and $1-\rho_\alpha$ being the probability of rejecting the draw.

$$\rho_\alpha = \min\left\{1, \frac{g_1(\alpha^{(t)}, \Omega^{(t-1)}}{g_2(\alpha^{(t-1)}, \Omega^{(t-1)})}\right\}$$

where: $g(\alpha,\Omega) = |\Omega|^{-(n+m+1)/2} \exp\left[-\frac{1}{2}\Im\{M(\alpha)\Omega^{-1}\}\right]$

with: - $\Im$: the trace of a matrix;

- $M(\alpha)$ is a squared matrix (sized up based on the number of cross-sections) with the ij$^{\text{th}}$ elements being: $(y_i - X_i\alpha_i)'(y_j - X_j\alpha_j)$.

Concerning Ω, a candidate $\Omega^{(t)}$ is generated from $f(\Omega|\alpha_{MLE}, Data)$ which is Wishart proposal density. The probability of accepting the candidate draw is therefore:

$$\rho_{\Omega} = \min\left\{1, \frac{g_3(\alpha^{(t)}, \Omega^{(t)}) / h_1(\Omega^{(t)} | \alpha_{MLE}, Data)}{g_1(\alpha^{(t)}, \Omega^{(t-1)}) / h_2(\Omega^{(t-1)} | \alpha_{MLE}, Data)}\right\},$$

while $1-\rho_{\Omega}$ will be the probability of rejecting the candidate draw.

XII. Estimating SUR model using DMC (Direct Monte Carlo)

The DMC approach is a method that presents some similarities with the MCMC and it is also used to compute Bayesian quantities using data generated from known models[13]. The literature provides enough evidence that the DMC approach is a valid method to compute Bayesian quantities (posterior (marginal) as well as predictive density functions). The DMC approach has the advantage of being easily applicable and useful in solving several problems with much less concerns per comparison to the MCMC approach. In comparison to the MCMC approach, in this sub-section, we provide some technicalities of the DMC algorithm applicable to a SUR model. We often refer to the SUR model because our MMM-DA estimates have been obtained using the Iterative SUR. Considering the parameters α and Ω, the DMC algorithm process can be summarized as follows (Zellner et al, 2008): (1) fixing the set of equations and samples to be generated; (2) sampling a (parameter obtained from a transformation applied on X, the matrix of unknown) and Ψ (the covariance matrix) using conditional posterior densities, $\{a^{(t)}, \Psi^{(t)}; t = 1,..., S\}$ with S being the samples to be generated; (3) transforming $\Psi^{(t)}$ into $\Omega^{(t)}$ [14]; and (4) sampling of the coefficient vector $\alpha^{(t)}$, using a normal density with:

- $\hat{\alpha}^{(t)} = \left\{X'(\Omega^{(t)^{-1}} \otimes S)X\right\}^{-1} X'(\Omega^{(t)^{-1}} \otimes S)y$ being the mean;
- and $\hat{\Omega}_{\alpha}^{(t)} = (X'(\Omega^{(t)^{-1}} \otimes S)X)^{-1}$.

From the conditional posterior densities, the DMC approach consists of several draws from the conditional inverse gamma density and repeating the process sequentially.

XIII. Loss Functions for estimating the MMM-DA

From a broad perspective, a loss function can be defined as a function used to assign a real number to a real number. The concept ‘loss’ is associated to the ‘regret’ felt when the target is not reached during an event. It captures the error orchestrated when the estimate is far

[13] Zellner A. and Tomohiro A.: ‘A Direct Monte Carlo Approach for Bayesian Analysis of the Seemingly Unrelated Regression Model’, Working paper, 2008.

[14]

away from the true value. Therefore, the key issue is to determine a consistent estimator that suits the loss to be experienced in case the target is not reached. As soon as the estimator is well identified, any type of loss function to be used aim at minimizing the expected loss observed when the target is not hit. Even though both loss functions are subject to a similar objective (minimization of the expected loss) it is imperative that the choice of the loss function be based on the kind of loss that will be faced in case of specific problem. Factors such as the socio-political environment matter most in this type of choice. Therefore, whenever a model is designed for policy guidance, knowing that a specific target is set, the loss functions to be used should be different. A major advantage linked to Bayesian loss functions is that experimental data used in the model does not on its own orient the final decision. The prior probability plays a larger role. The combination of both: (1) the prior probability; (2) the experimental data; and (3) the selected loss function; leads to the final decision obtained through the maximization of the subjective expected loss function (Savage, 1954). Referring to Savage's theory[15] on 'subjective expected utility', our objective function (the subjective expected loss function SE) can be presented as follows:

$$SE = \int_{-\infty}^{\infty} L(y)P(y)dy$$

where $L(y)$ is the loss function and y being the continuous random variable defined under a probability density function. As y can take different values, different subjective expected loss functions can therefore be set:

$$S_i = \int_{-\infty}^{\infty} L_i(y_i)P_i(y_i)dy_i$$

$$S_j = \int_{-\infty}^{\infty} L_j(y_j)P_j(y_j)dy_j$$

The preferred decision between the two will be driven by the lowest subjective expected loss. Taking convex combinations of different decisions could constitute a better option (Savage 1954).

Quadratic loss functions that are subject to a very popular usage are, in our view, not always appropriate when specific policy targets are pursued. Although quadratic loss functions are consistent with the mathematical principles based on their frequent reference to variances, they might undermine the definition of loss in certain cases. Whether the error is above or below the target, it generates the same loss provide that it has the same magnitude. This

[15] Savage L.J. (1954): 'Foundations of Statistics', New York, Wiley.

concept makes little sense whenever the target is 'inflation' or any similar macroeconomic indicator (i.e: GDP growth; poverty line; etc).

DATA COLLECTION

The data used in this study is secondary and collected from 1972 on a yearly basis. We have investigated 10 development sectors that constitute the contributors to the overall country's sales output[16]. The main sources of data collection are all official in addition to the links related forecasting models established. It is usually expected that link-forecasting units are most likely to provide clean and appropriate information. Aggregate output figures were the easiest to locate. Although data regarding input components as: Investment; Employment; Wages; etc; was much more difficult to locate especially in sectors without proper data warehousing systems. Generic sources like: (1) the SARB (South African Reserve Bank); (2) the IMF (International Monetary Fund); (3) the World Bank; or (4) the African Development Bank constituted the first choice. However the location of sectoral data including: sectoral production or demand; sectoral input utilisation; sectoral prices have remained the hardest to collect. Very little databases are currently available in order to provide such information necessary though. Ten development sectors of the South African economy are considered as cross sections in this study. Generally, though, above-mentioned sources provide data with some missing sectoral information making the model synchronisation process more difficult to achieve. To address the missing data problem, the analysis considers the option of solving for reduced form equations for output and uses the outcome for forecasts (Zellner and Israilevich, 2003). As it could be garnered from the number of participating sectors or forecasting links, the interest for this modelling exercise is very high although the disaggregated or sectoral data problem constitutes one of the major obstacles the present research has to face.

RESULTS DISCUSSION

Considering the criteria set earlier such as: RMSE; and MAE; it can be concluded that the MMM-DA constitutes a valid model for estimating and predicting the South African economy. MMM-DA predicts better than the benchmark (see graphs in the appendix).

[16] The sectors considered are the following: (1) Manufacturing; (2) Agriculture, Fishing and Forestry; (3) Construction and Buildings; (4) Mining; (5) Government; (6) Community services; (7) Transport and Telecommunication; (8) Financial services; (9) Wholesales, Retail, Catering and Accommodation; and (10) Electricity, Gas and Water.

Despite the low quality of data used, the model provides plausible outcomes. As some sectors such: (1) Electricity, Gas and Water; (2) Government; (3) Agriculture; and (4) Financial Services; have dynamics more complex than the simple profit maximizing rules, the results obtained are relatively uneasy to understand.

In order to assess the prediction ability of our model, we compare the forecasted series (sector's sales supply growth) with the actual one. That is the underlying concept driving the calculation of either the MAE or the RMSE. The MAE enforces the use of absolute values without any allusion to square losses while the RMSE consider the square or the error loss function. Different values of either MAE or RMSE can be obtained depending on the sample size or the sector considered. Sectors with large sales growth rates tend to have larger MAE or RMSE as compared to sectors with smaller sales growth rates. However, the average error obtained in our model accounts for less than 50 percent of the individual sector's sales growth rate. That is a clear indication of the model's ability to make predictions within a reasonable range. Related studies conducted on the US economy (Kim, 2006) have produced very similar percentage errors (average 50 percent). Regarding the early 2000s, the model presents much less forecasting ability, probably due to several unpredicted policy changes that occurred in the South African economy.

	ARLI (3)	MMM-DA (no shrinkage, Non-Bayesian)	MMM-DA (shrinkage, Non-Bayesian)	MCMC (Bayesian)	DMC (Bayesian)
RMSE	2.75	1.61	1.72	1.51	1.39
MAE	2.17	1.28	1.31	1.26	1.18

Table 1- RMSE and MAE[17]

[17] Formulas:

- $$MAE = \frac{1}{T}\sum_{t}^{T}\left|\hat{y}_t - y_t\right|$$
- $$RMSE = \sqrt{\frac{1}{T}\sum_{t}^{T}(\hat{y}_t - y_t)^2}$$

We have also performed simulations using MCMC (until 1500 iterations) and the MMM-DA still forecast much better than the ARLI (3).

The MMM-DA predicts much better than the benchmark (ARLI 3) and the use of shrinkage doesn't seem to bring much improvement. The DMC approach appeals much more as an easily applicable Bayesian technique as compared to the MCMC that requires more conditions and a more complicated algorithmic approach. Looking at different sectors considered, some of them have presented major forecasting improvements while using shrinkage techniques as opposed to other techniques. The use of shrinkage requires a supportive understanding of the interrelationship existing among sectors. When diversity is very large, shrinkage may not be the best option to take.

A set of restrictions have also been tested on the model in order to reduce the number of parameters. The test used (Wald test) suggests that the first two lagged variables of the sectoral sales supply ($S_{i,t-1}$ and $S_{i,t-2}$)) can be imposed a zero coefficient. A similar finding has been made earlier (Zellner, 2003).

CONCLUSION AND STUDY LIMITATION

The present research mainly focuses on the improvement of forecasting process through disaggregating with Bayesian versus non-Bayesian techniques and the use of shrinkage estimations (*à la* Stein). This research provides clearer evidence that a Marshallian Model constitutes a useful tool to understand and predict a country's growth throughout sectoral production activities. Despite the fact that the quality of some of the sectors' series remains questionable, first approximation can be taken from a full-fledged Marshallian Model for the South African economy. The empirical approach used in this research in terms of the Marshallian macroeconomic techniques and the shrinkage estimations is not without flaws. In some cases, as this research suggests, the use of shrinkage estimations does not always produce improvement in the model's prediction ability considering that sectors often have disparate dynamics. However, the MMA-DA when carefully used can be valuably introduced to guide output growth forecasting in multiple output sectoral units, as long as the data used in the analysis and the vector means is representative of the production process and can be compared to appropriate peer production units. As additional findings of this study, restrictions can validly be imposed on the first two lags of the MMM-DA without affecting the results. This conclusion has been supported in previous studies (Zellner et al, 2002). In addition to the existing literature, the present research has provided some extensions to the Marshallian modelling process by introducing: (1) an entry cost; (2) the human capital aspect of production (labor effectiveness); (3) a broader aspect of the sales demand function; and (4) the foreign sector.

This study opens new horizons for further researches in forecasting models including more detailed leading indicators and probably deeper disaggregation (i.e. regional disaggregation). In reaction to some valuable suggestions made during the presentation of this model, we carefully consider expanding the exercise much further by including more features to the research. In fact, additions such as: (1) the dynamic of inventories; (2) the capital market; (3) the entry of new sectors (Schumpeterian innovations); (4) the distinction between skilled and unskilled labor force; (5) the use of a generalised production function instead of restricting the process to the use of a Cobb-Douglas production function; etc; might increase the prediction ability of the model. In connection with what we discussed earlier regarding the optimality of a maximum likelihood estimator iterated while using SUR, when the loss function is defined as a zero-one loss function, the modal value obtained using SUR is optimal. It appeals to us that a zero-one loss function seems very appropriate in policy making process since policymakers act according to specific targets. When the target is missed, there is a big loss, and vice versa. However, in some further studies this issue will be discussed more thoroughly. When one decide to go Bayesian, the use of DMC seems more appealing and easily applicable as compared to the MCMC approach.

REFERENCES

Bloom, D.E., D. Canning, and J. Sevilla. 2005. “Health and Economic Growth: Reconciling the Micro and Macro Evidence.” Harvard School of Public Health, MA. Processed.

Dey, D.K, Ghosh, M. and Strawderman, W.E.(1999). On Estimation with Balanced Loss Functions. Statist. Probab. Lett., 45, 97 – 101

De Alba, E. and Zellner, A. (1991). Aggregation, Disaggregation, Predictive Precision and Modelling. Manuscript, H.G.B. Alexander Research Foundation, Graduate School of Business, University of Chicago.

ESPD – ECA November 2005 Report: “An Applied Manual for use by African /ministries of Finance and Economic Development.

Green, E. and Strawderman, W.E. (1996). A Bayesian Growth and Yield Model for Slash Pine Plantations. J. Appl. Stat., 23, 285-299

Leonard, T. and Hsu, J.S.J.(2001). Bayesian Methods. Cambridge: Cambridge University. Press.

Kibambe, J. and Koch, S.(2005). Improving policy implementation by the use of efficiency models: An application of DEA on public hospitals. Working Paper, University of Pretoria.

Kim, K.H. (2006). Empirical Evidence of Forecasting Improvements from Disaggregating the US Economy. Working Paper, University of Chicago.

Lindley, D.V. and Smith, A.F.M.(1972). Bayes Estimates for the Linear Model. J. R. Stat. Soc. Ser. B, 34, 1-41.

Muth, J.F. (1961). Rational Expectations and the Theory of Price Movements. Econometrica, volume 29.

Rossi E.P., Allenby G.M. and McCulloch R. (2005) Bayesian Statistics and Marketing. Wiley Series in Probability Statistics.

Sclove, S.L. (1968). Improved Estimators for Coefficients in Linear Regression. J. Amer. Statist. Assoc. 63, 596-606.

Stein, C. (1962). Confidence Sets for the mean of a Multivariate Normal Distribution. J. R. Stat. Soc. Ser B 24.

Van der Merwe, A. and Viljoen, C. (1998). Bayesian Analysis of the Seemingly Unrelated Regression Models. ms., Department of Mathematical Statistics, University of the Free State, Bloemfontein. South Africa.

William, H.H. (1954). The Theory of Collective Bargaining. The Free Press Glencoe, New York.

Zarnowitz, V. (1986). The Record and Improvability of Economic Forecasting. Economic Forecastings, 3, 22-31.

Zellner A. (1986a). Bayesian Estimation and Prediction Using Asymmetric Loss Functions. J. Amer. Statist. Assoc. 81, 446-451.

Zellner, A. and Vandaele, W.A. (1974). Bayes-Stein Estimators for k-means, Regressions and Simultaneous Equation Models. Amsterdam. 627-653.

Zellner, A. (2002) Bayesian Shrinkage Estimates and Forecasts of Individual and Total or Aggregate Outcomes. H.G.B. Alexander Research Foundation. Graduate School of Business. University of Chicago.

Zellner, A. and Guillermo I. (2003). The Marshallian Macroeconomic Model: A Progress Report. H.G.B. Alexander Research Foundation. Graduate School of Business. University of Chicago.

Zellner, A. and Tobias, J. A note on Aggregation, Disaggregation and Forecasting Performance. H.G.B. Alexander Research Foundation. Graduate School of Business. University of Chicago.

Zellner, A. and Tomohiro A. (2008) A Direct Monte Carlo Approach for Bayesian Analysis of the Seemingly Unrelated Regression Model. University of Chicago.

APPENDIX A: Deriving the RFE (Continuous time)

Considering the three equations: (1) sales supply; (2) sales demand; and (3) entry/exit; the final RFE-DA have been derived as follows:

$$\frac{\dot{S}_S}{S_S} = \theta_1 \frac{\dot{A}}{A} + \left(\frac{\dot{\Im}}{\Im} + \frac{\dot{N}}{N} \right) + \theta_2 \frac{\dot{P}_Q}{P_Q} + \theta_3 \frac{\dot{w}}{w} + \theta_4 \frac{\dot{r}}{r} + \sum_{l=1}^{T} \sigma_l \frac{\dot{P}_l}{P_l} + \theta_5 \frac{\dot{z}}{z} + \theta_6 \frac{\dot{\Gamma}}{\Gamma}$$

(2.2.4)

$$\frac{\dot{S}_D}{S_D} = (1-\Delta)\frac{\dot{P}_Q}{P_Q} + \frac{(\dot{\Re D})}{(\Re D)} + (\lambda_1 + \Delta)\frac{\dot{P}_Q^e}{P_Q^e} + \chi_{j1}\frac{\dot{Y}_d}{Y_d} + \chi_{j2}\frac{\dot{WY}}{WY}$$

(2.3.4)

$$\frac{\dot{N}}{N} = C_E (S_S - \pi^e)$$

(2.6.1)

Equating equations 2.2.4 and 2.3.4:

$$\theta_1 \frac{\dot{A}}{A} + \left(\frac{\dot{\Im}}{\Im} + \frac{\dot{N}(\Gamma)}{N(\Gamma)} \right) + \theta_2 \frac{\dot{P}_Q}{P_Q} + \theta_3 \frac{\dot{w}}{w} + \theta_4 \frac{\dot{r}}{r} + \sum_{l=1}^{T} \sigma_l \frac{\dot{P}_l}{P_l} + \theta_5 \frac{\dot{z}}{z} + \theta_6 \frac{\dot{\Gamma}}{\Gamma} = (1-\Delta)\frac{\dot{P}_Q}{P_Q} + \left(\frac{\dot{\Re}}{\Re} + \frac{\dot{D}}{D} \right) + (\lambda_1 + \Delta)\frac{\dot{P}_Q^e}{P_Q^e}$$
$$+ \chi_1 \frac{\dot{Y}_d}{Y_d} + \chi_2 \frac{\dot{WY}}{WY}$$

(A.1)

Replacing $\frac{\dot{N}}{N}$ in equation A.1 by equation 2.6.1:

$$\frac{\dot{P}_Q}{P_Q} = \frac{C_E}{\theta_2 - 1 + \Delta}.\pi^e - \frac{C_E}{\theta_2 - 1 + \Delta}.S_S + \frac{1}{\theta_2 - 1 + \Delta}.\frac{\dot{D}}{D} + \frac{(\lambda_1 + \Delta)}{\theta_2 - 1 + \Delta}.\frac{\dot{P}_Q^e}{P_Q^e} - \frac{\theta_1}{\theta_2 - 1 + \Delta}.\frac{\dot{A}}{A} - \frac{\theta_3}{\theta_2 - 1 + \Delta}.\frac{\dot{w}}{w}$$
$$- \frac{\theta_4}{\theta_2 - 1 + \Delta}.\frac{\dot{r}}{r} - \frac{\sum_{l=1}^{T} \sigma_l}{\theta_2 - 1 + \Delta}.\frac{\dot{P}_l}{P_l} - \frac{\theta_5}{\theta_2 - 1 + \Delta}.\frac{\dot{z}}{z} - \frac{\theta_6}{\theta_2 - 1 + \Delta}.\frac{\dot{\Gamma}}{\Gamma} + \frac{\chi_1}{\theta_2 - 1 + \Delta}.\frac{\dot{Y}_d}{Y_d} + \frac{\chi_2}{\theta_2 - 1 + \Delta}.\frac{\dot{WY}}{WY}$$

(A.2)

Plugging A.2 and 2.6.1 into 2.2.4can be written in a simplified form with equation 2.3.4 plugged into equation 2.2.4 in order to obtain the following RFE-DA for price and sales supply:

$$\frac{\dot{S}_{Si}}{S_{Si}} = \theta'_{0i} + \theta'_{1i} S_{Si} + \theta'_{2i} \frac{\dot{D}_i}{D_i} + \theta'_{3i} \frac{\dot{A}_i}{A_i} + \theta'_{4i} \frac{\dot{w}_i}{w_i} + \theta'_{5i} \frac{\dot{r}}{r} + \theta'_{6i} \frac{\dot{z}_i}{z_i} + \theta'_{7i} \frac{\dot{Y}_d}{Y_d} + \theta'_{8i} \frac{\dot{WY}}{WY} + \theta'_{9i} \frac{\dot{\Gamma}}{\Gamma} + \sum_{l=1}^{T} \wp_{il} \frac{\dot{P}_{il}}{P_{il}}$$

$$\frac{\dot{P}_{Qi}}{P_{Qi}} = \sigma'_{0i} + \sigma'_{1i} S_{Si} + \sigma'_{2i} \frac{\dot{D}_i}{D_i} + \sigma'_{3i} \frac{\dot{A}_i}{A_i} + \sigma'_{4i} \frac{\dot{w}_i}{w_i} + \sigma'_{5i} \frac{\dot{r}}{r} + \sigma'_{6i} \frac{\dot{z}_i}{z_i} + \sigma'_{7i} \frac{\dot{Y}_d}{Y_d} + \sigma'_{8i} \frac{\dot{WY}}{WY} + \sigma'_{9i} \frac{\dot{\Gamma}}{\Gamma} + \sum_{l=1}^{T} \wp_{il} \frac{\dot{P}_{il}}{P_{il}}$$

with: $\sum_{l=1}^{T} \wp_{il} \frac{\dot{P}_{il}}{P_{il}} = \frac{(\lambda_1 + \Delta)}{\theta_2 - 1 + \Delta} . \frac{\dot{P}^e_{Q_{it}}}{P^e_{Q_{it}}} - \frac{\sum_{l=1}^{T} \sigma_l}{\theta_2 - 1 + \Delta} . \frac{\dot{P}_l}{P_l}$

APPENDIX B

B.1. Model fitness results

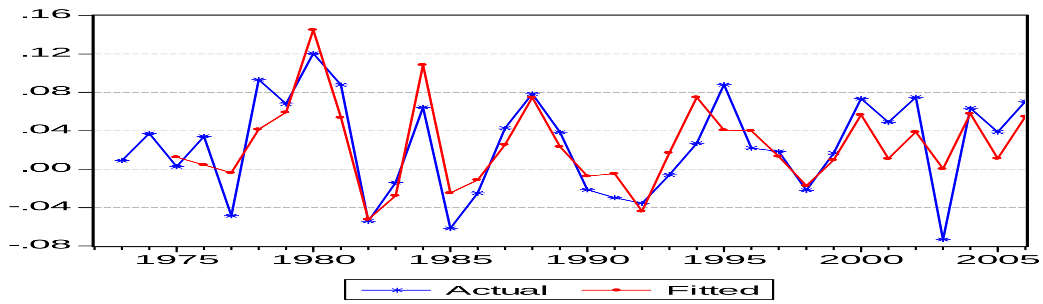

Figure 2: Manufacturing

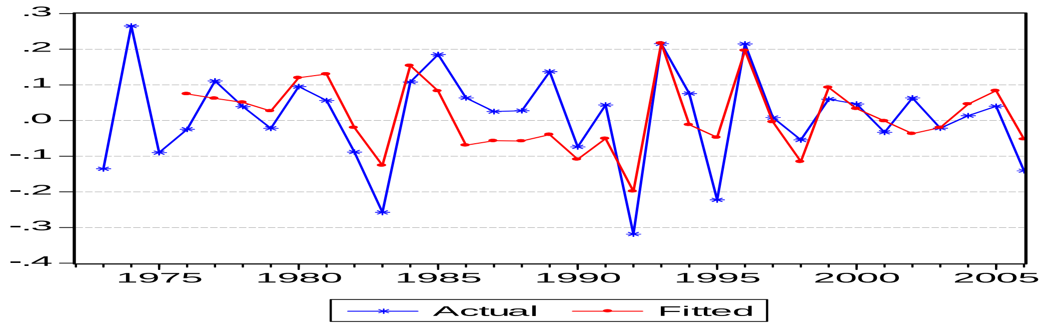

Figure 3: Agriculture

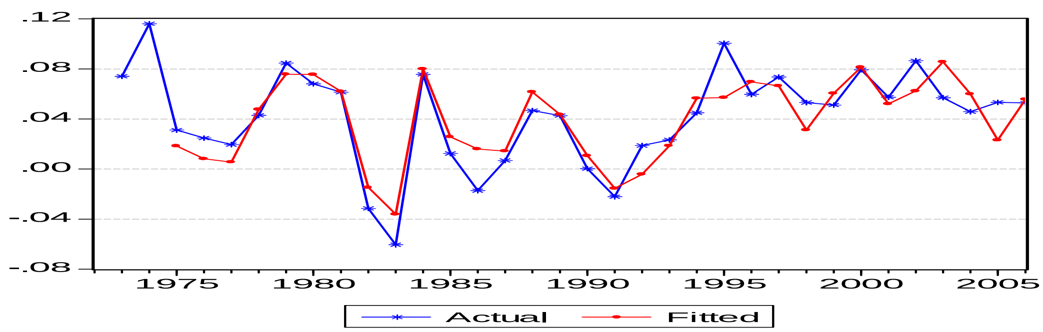

Figure 4: Transport

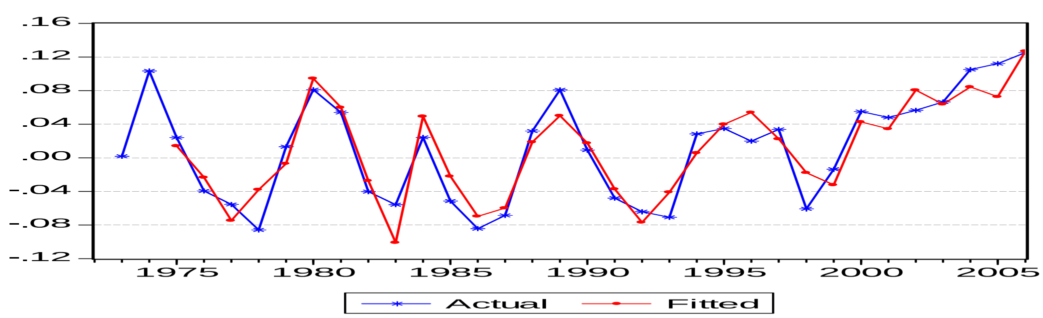

Figure 5: Construction

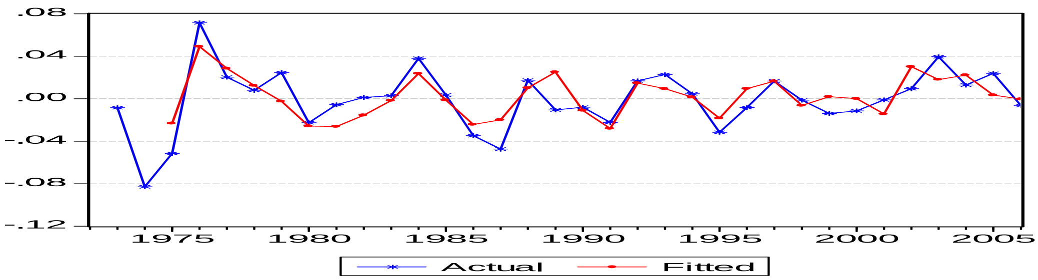

Figure 6: Mining

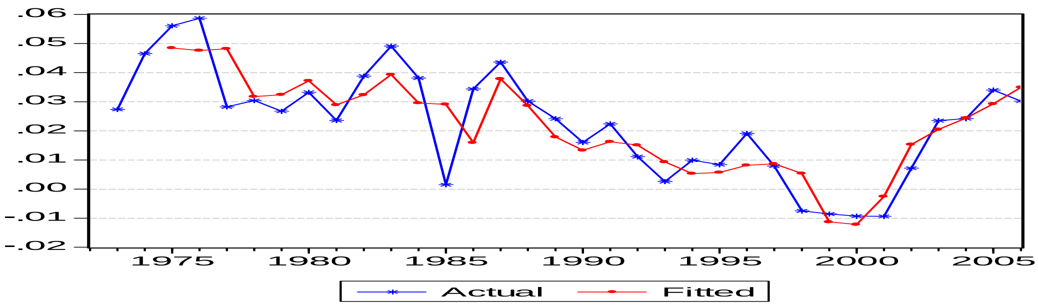

Figure 7: Government

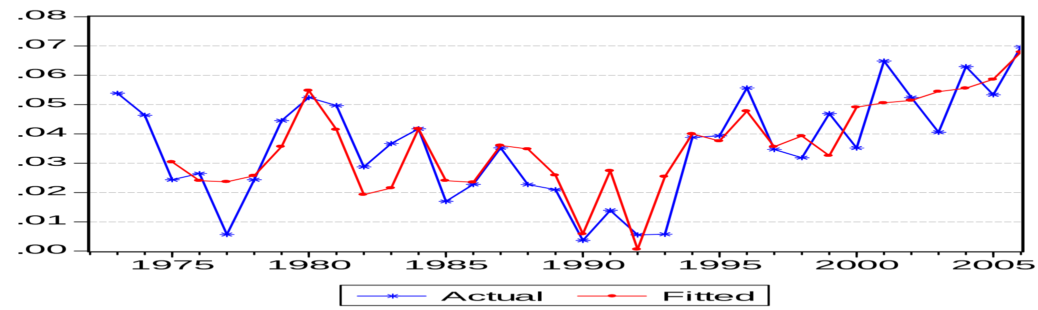

Figure 8: Community services

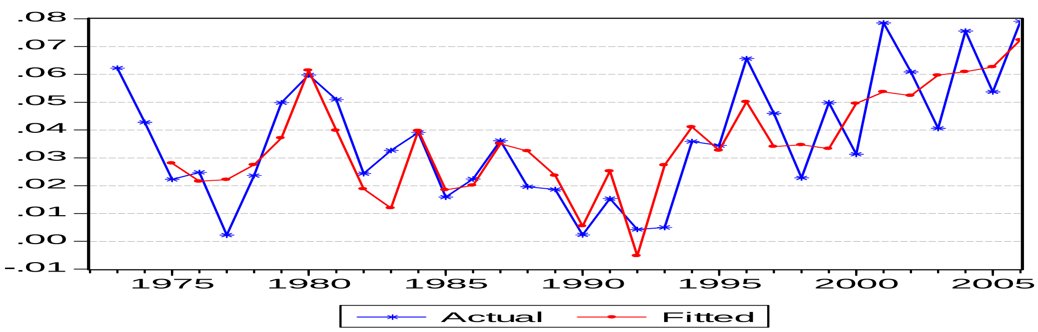

Figure 9: Financial sector

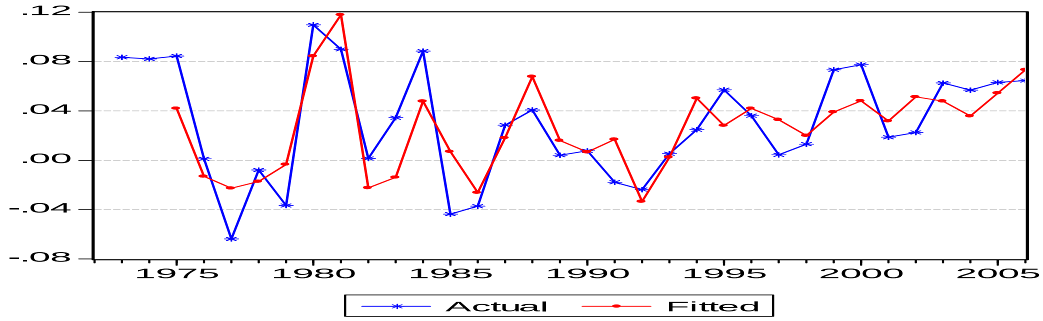

Figure 10: Wholesales

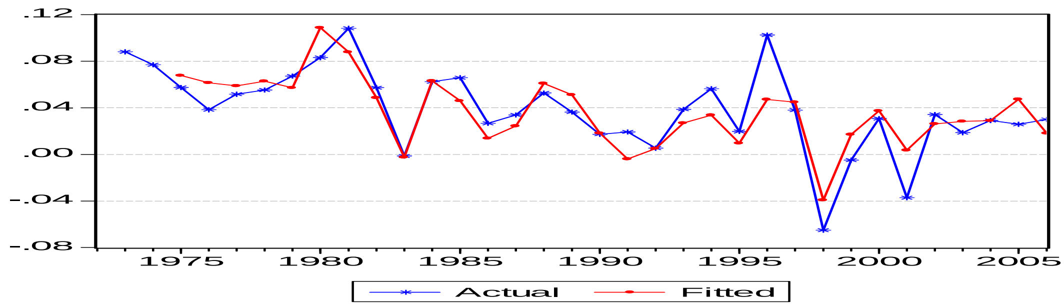

Figure 11: Electricity

B.2. Model's Prediction Ability: Actual versus Predictions (Forecasts)

In this section, results of the one-year-ahead forecast are provided for individual sectors assessing the forecasting performance of the MMM-DA. Predictions are conducted from 1995 until 2006. Based on the differences between predictions and actual values, the RMSE and the MAE are computed.

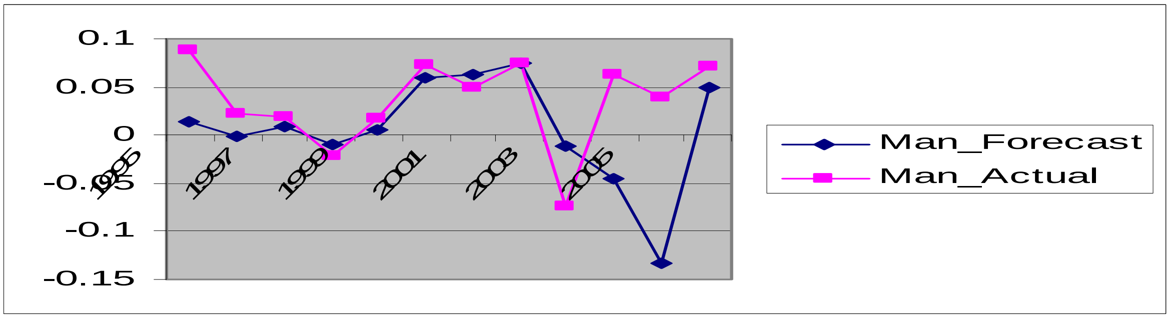

Figure 12: Manufacturing

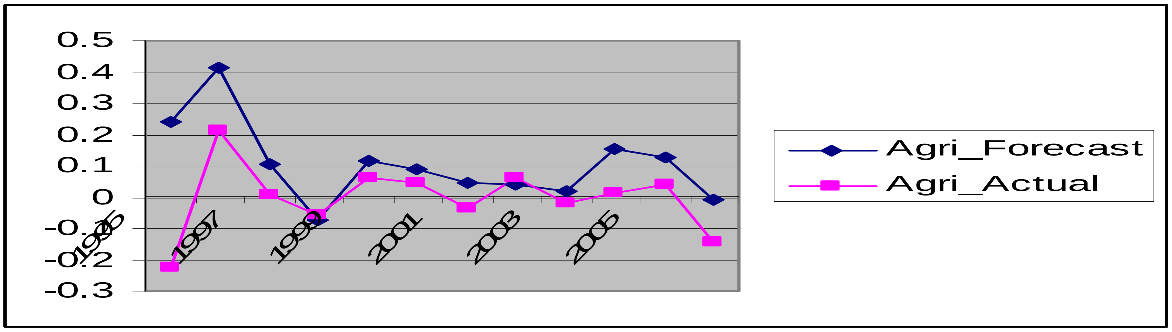

Figure 13: Agriculture

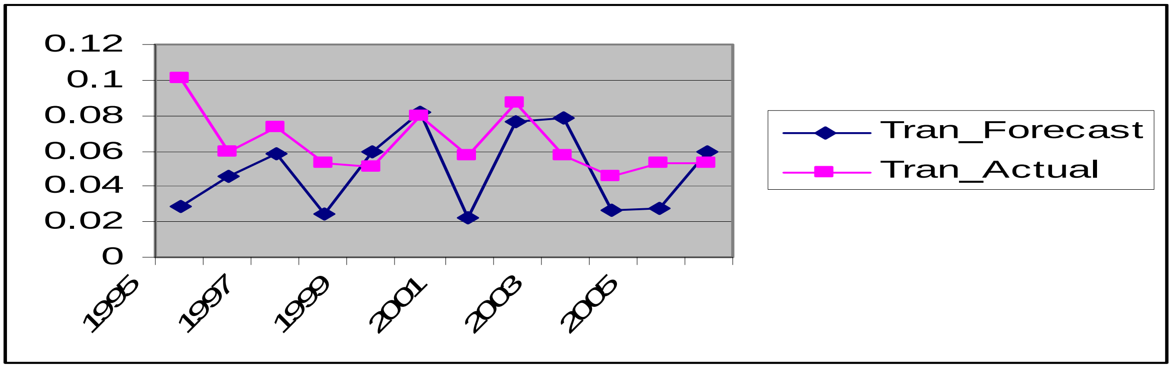

Figure 14: Transport

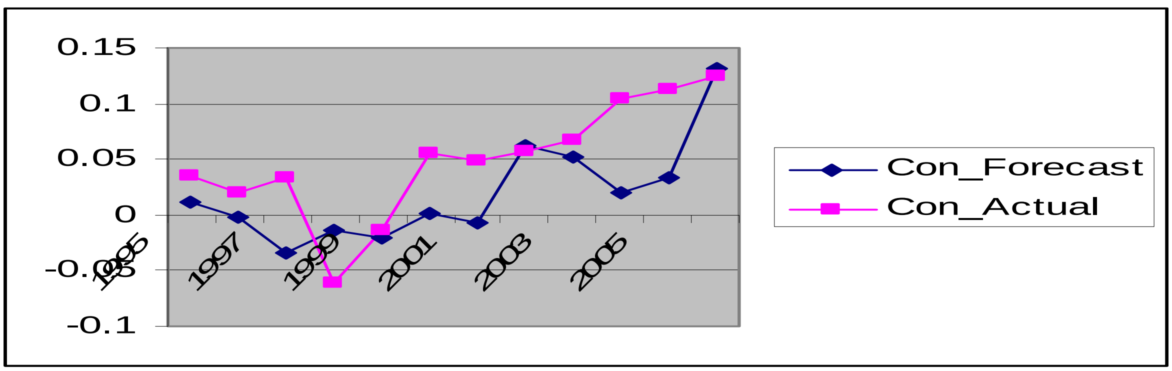

Figure 15: Construction

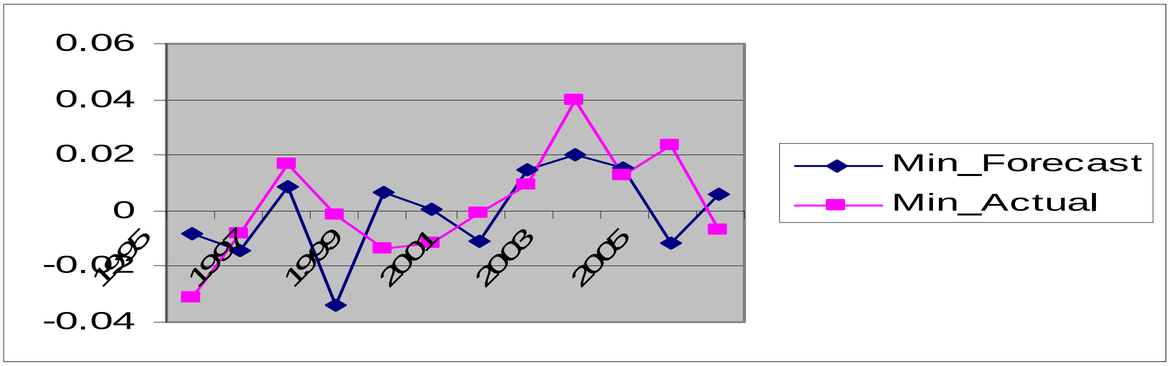

Figure 16: Mining

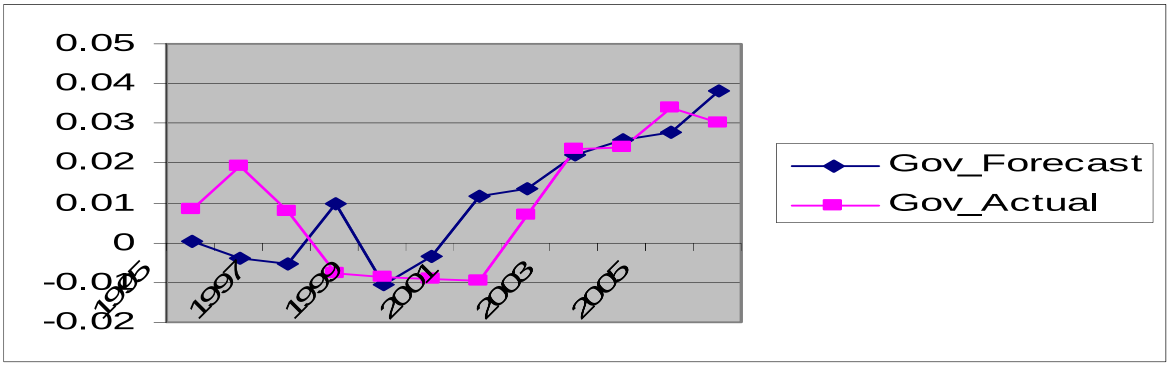

Figure 17: Government

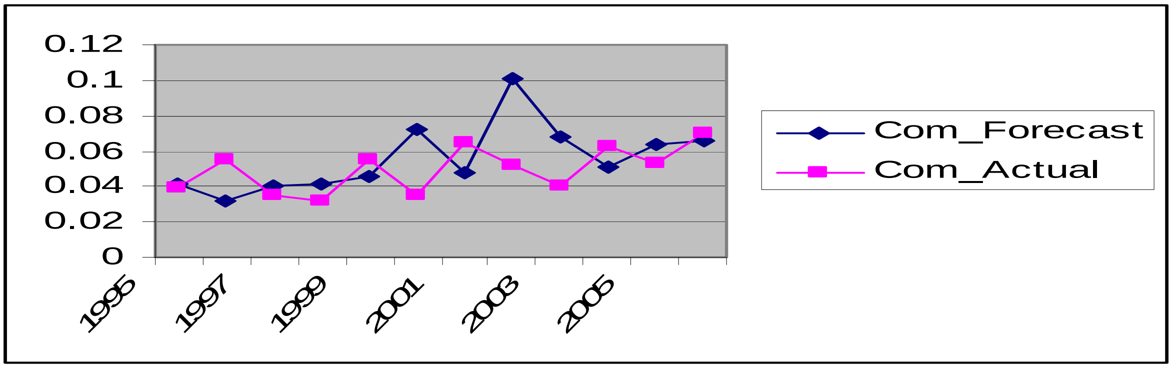

Figure 18: Community services

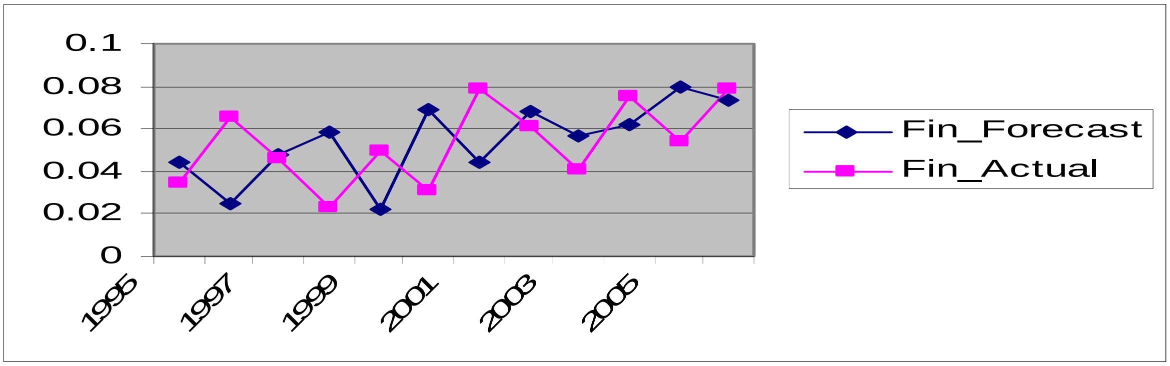

Figure 19: Financial services

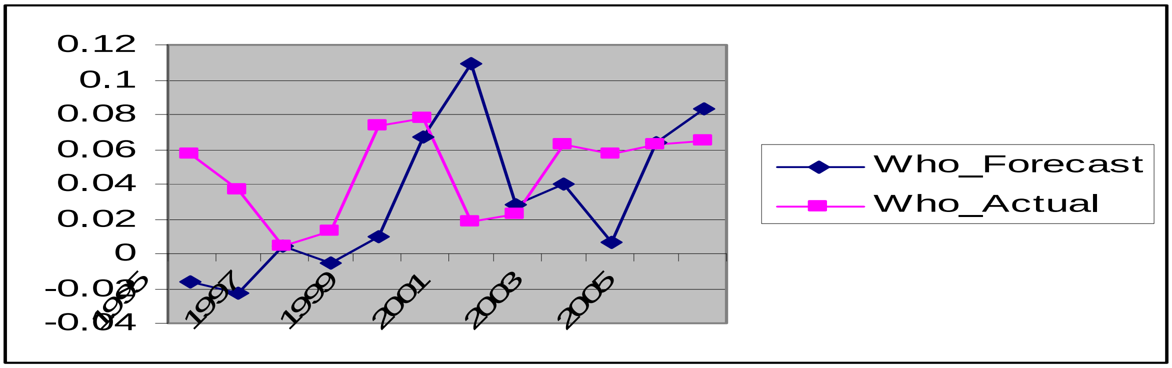

Figure 20: Wholesales

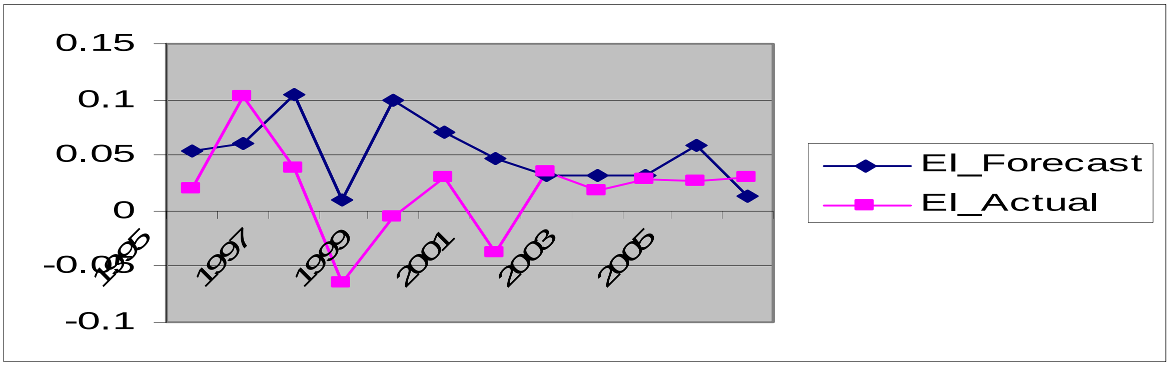

Figure 21: Electricity

B.3. Model fitness using complete shrinkage

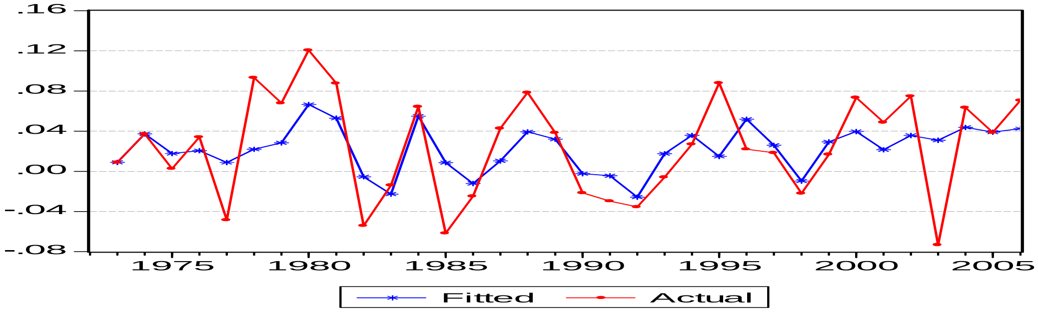

Figure 22: Manufacturing

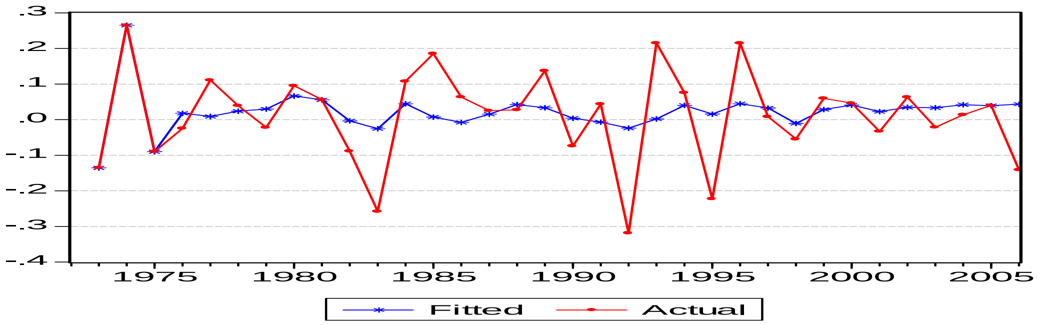

Figure 23: Agriculture

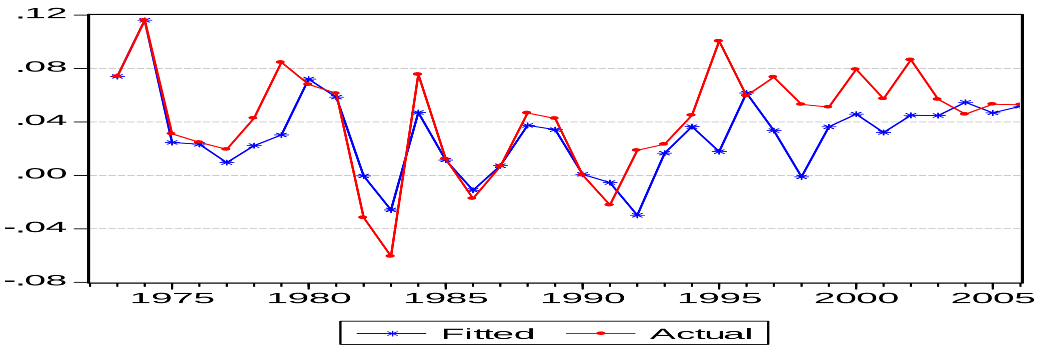

Figure 24: Transport

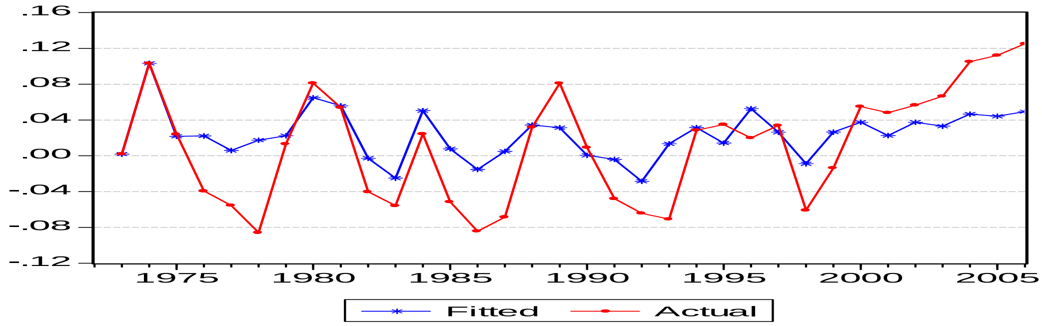

Figure 25: Construction

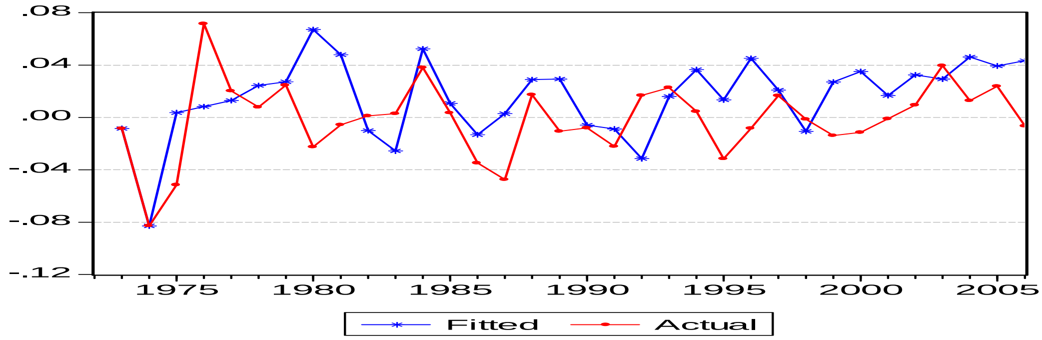

Figure 26: Mining

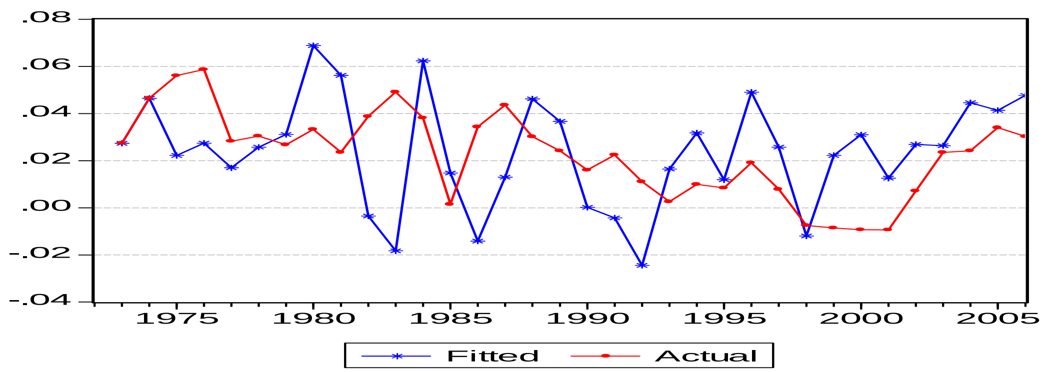

Figure 27: Government

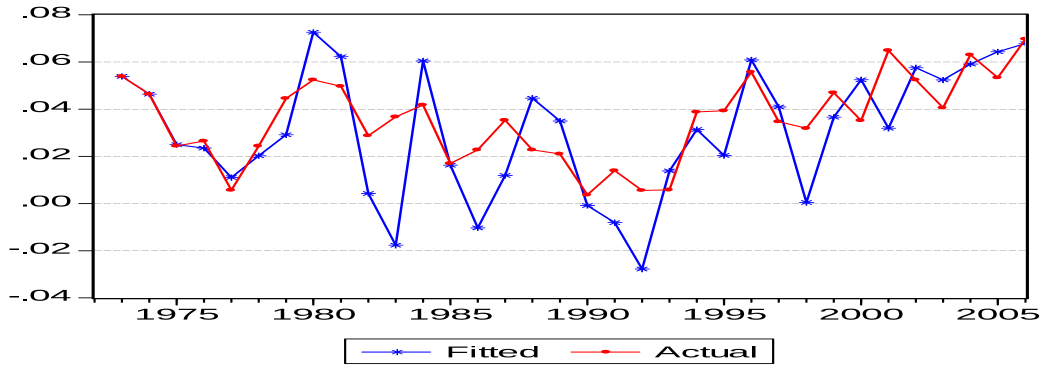

Figure 28: Community

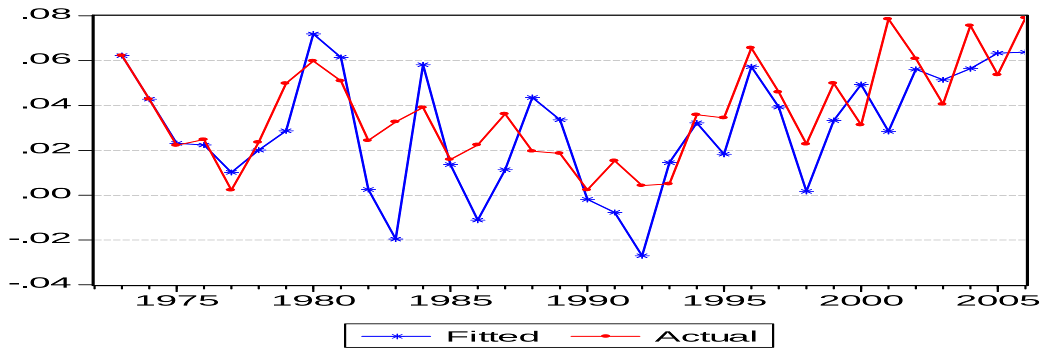

Figure 29: Financial

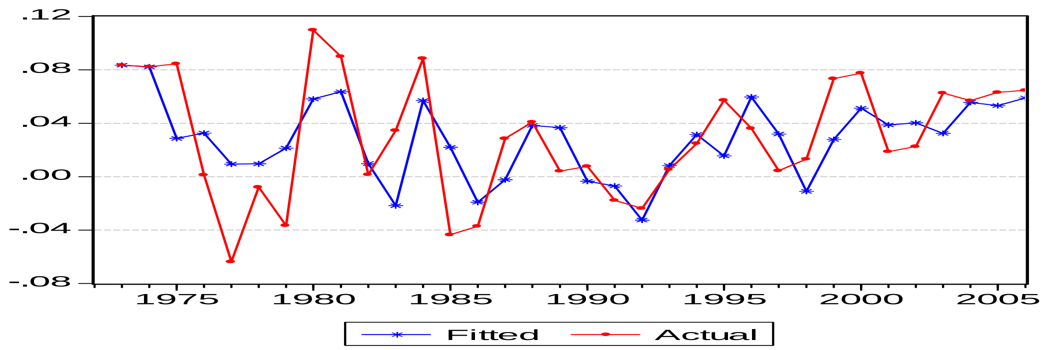

Figure 30: Wholesales

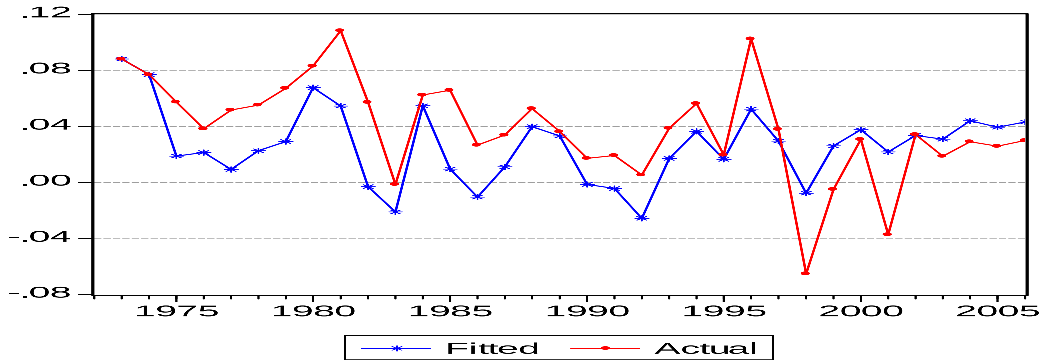

Figure 31: Electricity

B.4. Predictions (forecasting) using shrinkage technique

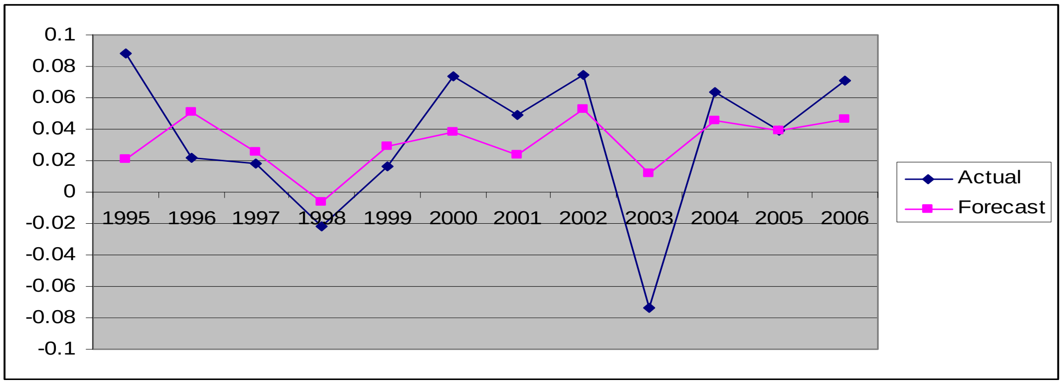

Figure 32: Manufacturing

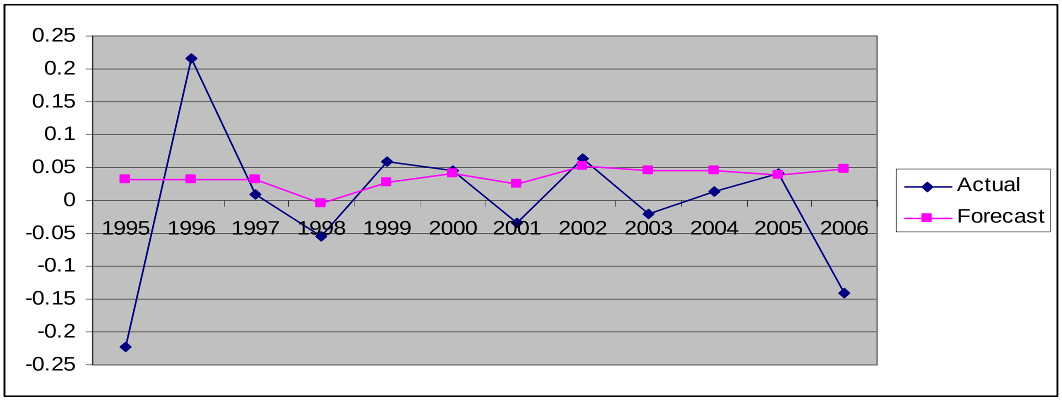

Figure 33: Agriculture

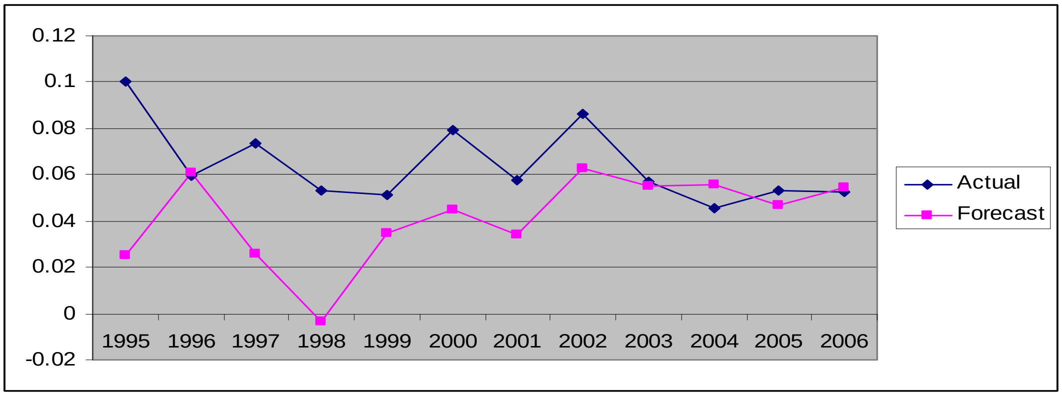

Figure 34: Transport

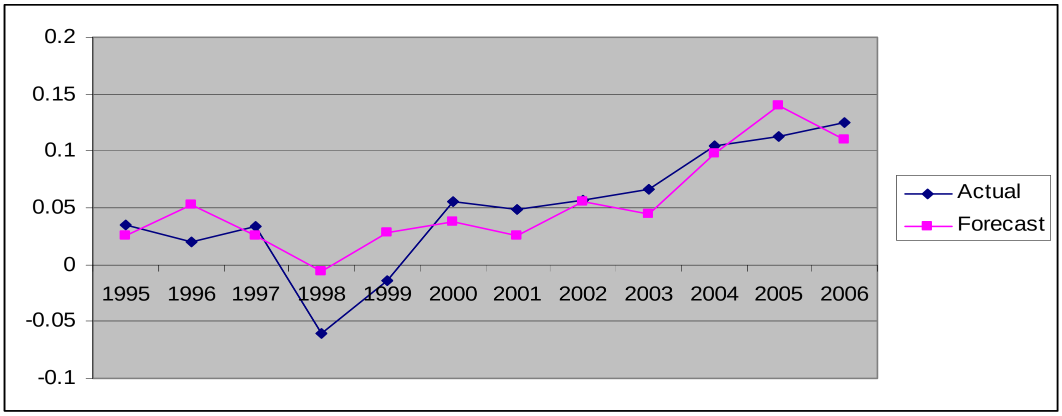

Figure 35: Construction

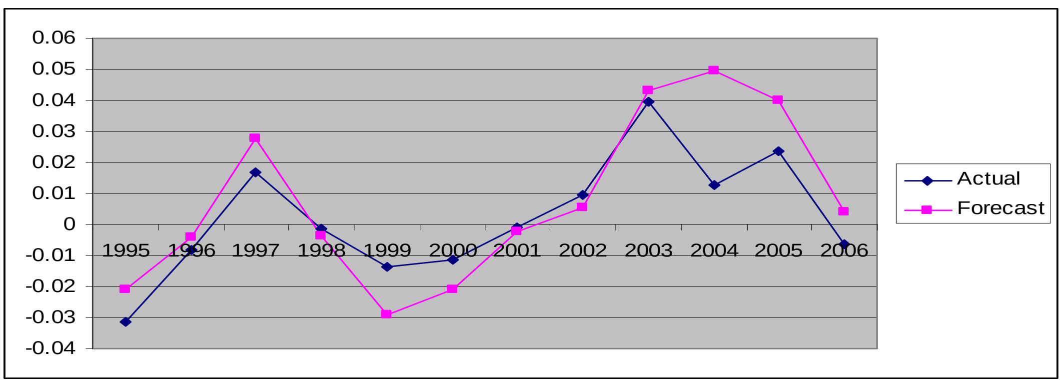

Figure 36: Mining

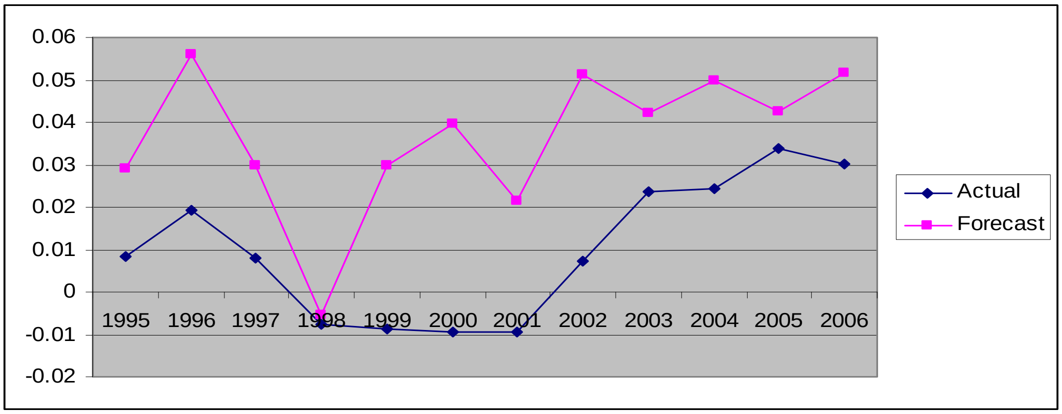

Figure 37: Government

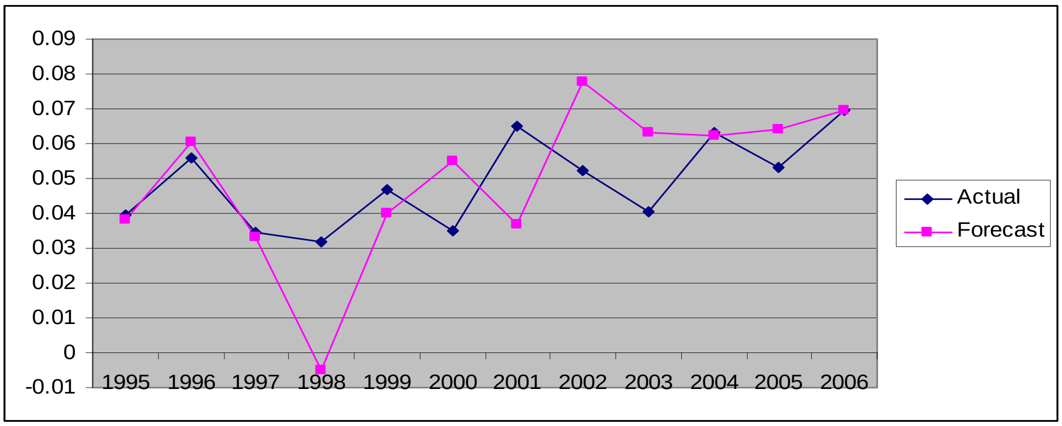

Figure 38: Community

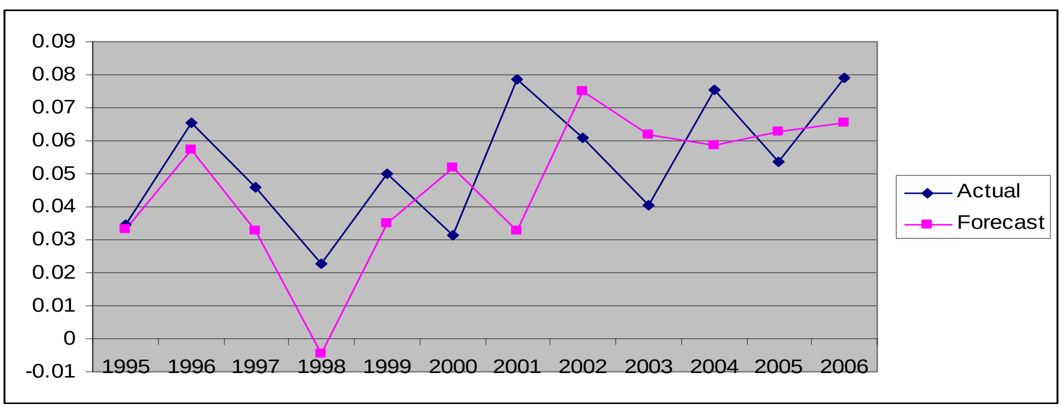

Figure 39: Financial services

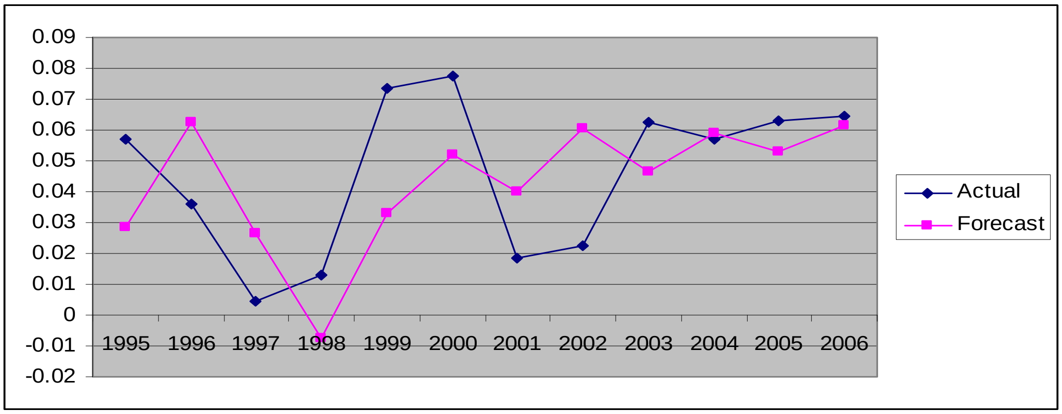

Figure 40: Wholesales

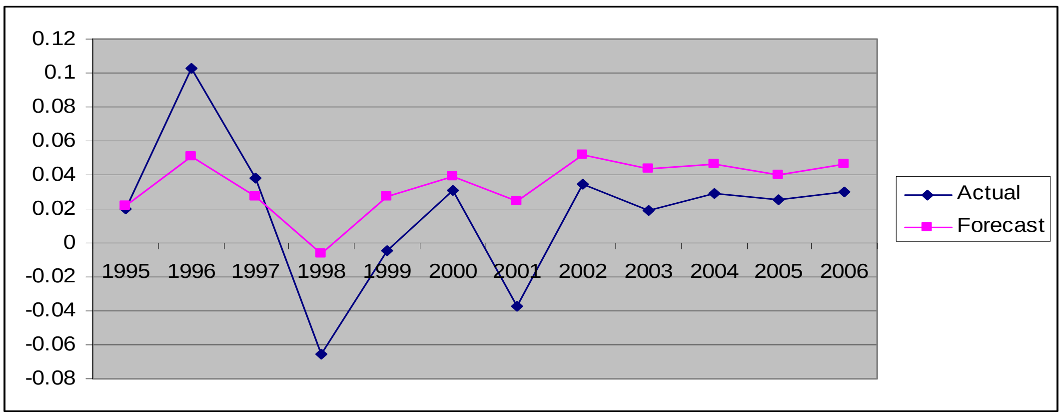

Figure 41: Electricity