A DISTRIBUTIONAL COMPARISON OF SIZE-BASED PORTFOLIOS ON THE JSE

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ABSTRACT

This paper examined the probability distributions of smaller capitalisation and larger capitalisation portfolios. We fitted both Normal and two component mixtures of Normal distributions to the portfolios. The mixture distributions were then related to a two-state model, revealing several differences between the behaviour of the smaller and larger capitalisation portfolios.

Specifically, we fitted scale mixtures of Normal distributions to the portfolios' monthly log returns. Parameters were estimated by means of the Expectation Maximisation (EM) algorithm. Using Chi-squared goodness of fit tests, we tested the adequacy of fit and compared this to the fit achieved by the Normal distribution. It was found that the scale mixtures outperformed the Normal distribution in all cases.

Using the estimates from the fitted scale mixtures, we compared the portfolios in terms of a two state-model. The two-state model hypothesizes that equities move between a low and high volatility state, such that each state is represented by the low and high volatility components of a mixture distribution. By way of

F-test, we showed that the smaller and larger capitalization portfolios were similarly risky during periods of low volatility, but that the smaller capitalization portfolios were riskier when in the high volatility state. Also, the larger capitalization portfolios were in the higher volatility state more often than the smaller capitalization portfolios.

It should be clear from the above that this paper contributes to both the literature on asset return distributions and the debate concerning size-effects. It is thus inline with the recent work of Bahng (2004), who modelled the Swiss Market Index using a mixture of Normal distributions in a structural break framework, as well as van Rensburg and Robertson's (2003) paper on cross-section of JSE returns. It further ties in with Bundoo's (2006) study of size and value effects on the Stock Exchange of Mauritius.

INTRODUCTION

The assumption of Normality is pervasive within the fields of econometrics and finance, forming the basis of inference in OLS models as well as the foundation of the Black and Scholes model. The Normal distribution, however, fails to capture the characteristic heavy-tails associated with equity returns.

As an alternative, this paper used a two-component scale-mixture of Normal distributions. Forming smaller capitalization and larger capitalization portfolios, mixtures were fitted and compared to the Normal distribution. This showed that the mixture distributions provided a significant improvement over the Normal distribution. We further compared the distributions of the portfolios, adding to the debate around size-effects.

LITERATURE REVIEW

Bachelier's early research, in 1900, theorized that equity returns were distributed Normally (Tucker 1990). Later studies, however, found the assumption of Normality ill-suited. In South Africa, Clark and Troskie (2006) found that their sample of thirty three large capitalization JSE-listed shares were non-Normal and displayed excessive kurtosis with negligible skewness.

The failure of the Normal distribution resulted in research into two broad categories of alternative return processes (Bahng 2004, Tucker 1990). The first approach, into which this study falls, is the time-independent distributional modelling approach. Under this approach, various return processes have been suggested. Tucker (1990) compares four of these alternate processes, namely the stable Paretian, the Student process, a mixed diffusion-jump model and the Normal mixture model.

The second approach encompasses time-dependent models (Bahng 2004, Tucker 1990), which consist mostly of ARCH-type models. This has been the preferred approach in recent studies (Bahng 2004, Tucker 1990), largely due to ARCH-type models' ability to explain volatility clustering. As an example, Samouilhan (2007) uses a Component Generalized ARCH model to study the persistence of volatility clustering among different sectors of JSE listed companies, finding a material difference among the behaviour of conditional volatility between sectors.

Bahng (2004), however, demonstrates how a mixture of Normal distributions may be used as an alternative to ARCH. Using Goldfeld and Quandt's two-period structural break analysis, Bahng (2004) divides a time-series of the Swiss Market Index (SMI) returns into sub-periods, modelling each sub-period as a Normal distribution. The result for the entire time-series is a mixture of Normals.

Alexander (2001:297) suggests a simpler two-component scale-mixture of Normals, hypothesizing that the markets switches between states of high and low volatility. The model appears to be based on the work of Kon (1984), who suggested a mixture of three Normal distributions with one non-informational component, a firm-specific informational component and a market-wide informational component. A two-component scale-mixture of Normal distributions concurs, however, with the empirical findings of Tucker (1990).

Separate from the issue of non-Normality, there remain questions about apparent size effects. Cuthbertson and Nitzsche (2004:435) cite research indicating that the small-firm effect had, for the US market, disappeared after the 1980s. This view is contended by Elfakhania and Wei (2003) and Zepp (2003) who, respectively, find evidence of a combined size-price effect on the Canadian market and a small-firm effect for US water utility companies.

For developing markets, recent work appears to be in favour of a small-firm effect. Using a Fama-French three factor model, Bundoo (2006) shows that there exist significant size and value effects on the Stock Exchange of Mauritius (SEM). For South Africa, van Rensburg and Robertson's (2006) style-based two factor model lends support to the small-firm effect. Their model identified a significant negative relationship between size and equity returns.

RESEARCH METHODOLOGY

This section details our research methodology. First we describe the data and then we specify the procedure used to form our larger and smaller capitalization portfolios. Following this we mention the return calculations, distribution fitting and goodness of fit testing. The discussion on distribution fitting also provides some background to Normal mixtures.

Dataset

The dataset, purchased from Johannesburg Securities Exchange Limited, consisted of monthly price, market capitalization and volume data for companies listed during the period January 1990 till December 2005. There were a total of 366 shares, with 132 listed for the entire period. Where series had missing values, these were replaced with the last observed record. Dividends were not included in the dataset.

Delisted firms also were not included and no attempt at correcting for survivorship bias was possible. While Pawley (2006) notes significant survivorship bias within the South African unit trust industry, the Canadian study by Elfakhani and Wei (2003) find that survivorship bias was only significant at the 11% level. Bundoo (2006) cites further studies which support the notion that survivorship bias is often overemphasized. The effect of survivorship bias is therefore unclear.

Portfolio Creation

We selected two mutually exclusive groups of shares. The smaller capitalization group consisted of the twenty smallest capitalization shares, with positive trading volume, at the end of January 1990. As an additional constraint, we specified that the share price had to be less than R2.50. This prevented the inclusion of high priced shares with a low number of shares in issues. The large capitalization group consisted of the twenty largest capitalization shares, at the end of January 1990, with price greater than R2.50 and positive trading volume.

For each group, we created an equally weighted portfolio. We also created a log weighted portfolio for the smaller capitalization group and an inverse log weighted portfolio for larger capitalization group. Log (inverse log) weighting resulted in assigning more (less) weight to the smaller (larger) shares in the smaller (larger) capitalization group. This made these portfolios more sensitive to size effects that the equally weighted portfolios.

We therefore had the following four portfolios:

1. An equally weighted smaller capitalization portfolio, or EWS-portfolio.
2. An equally weighted larger capitalization portfolio, or EWL-portfolio.
3. A log weighted smaller capitalization portfolio, or LWS-portfolio.
4. An inverse log weighted larger capitalization portfolio, or IWL-

portfolio.

The paper will, henceforth, refer to the above portfolios using the stated abbreviations.

Return Calculation

Month-on-month log return series were calculated for each of the four portfolios. Each return series contained 191 monthly observations, starting February 1990.

Distribution Fitting

Both Normal and two-component scale-mixture of Normal distributions were fitted to the portfolios. While fitting Normal distributions via maximum likelihood estimation (MLE) is trivial, the procedure for the mixture distributions requires some explanation.

A mixture of Normals is simply a probability weighted average of Normal distributions. We, therefore, have to estimate the weight, mean and variance associated with each Normal distribution in the weighted average. Relating this to the Alexander's model (2001:297), we see that mean return and return volatility must be estimated for each of the two states, as well the probability of each state occurring. In the terminology of mixture models, each Normal distribution is a component in a two-component mixture of Normals. The probability associated with observations coming from either state, or component distribution, is the component weight.

Unlike Bahng (2004), we did not divide the data into different sub periods and could not assign observations to particular component distributions. There was, thus, no observable indicator marking observations as belonging to a specific component. We, however, assume such an indicator variable exists. This unobservable indicator together with the observed returns forms a hypothetical complete dataset. The estimation then reduces to an incomplete-data problem (McLachlan & Peel 2000:19), and we can use the Expectation Maximization (EM) algorithm to estimate the model parameters.

Goodness of Fit Testing

Goodness of fit, for both Normal and mixture distributions, was determined using Pearson's Chi-squared goodness of fit test.

RESULTS

In what follows we report basic descriptive statistics, the results from distribution fitting and goodness of fit testing.

Summary Statistics

	EWS-portfolio	EWL-portfolio	LWS-portfolio	IWL-portfolio
Mean	0.01565	0.00475	0.01517	0.004818
Standard Deviation	0.08130	0.07182	0.07348	0.07499
Excess Kurtosis	8.78461	1.49542	5.36520	1.42655
Skewness	-1.44095	-0.66958	-0.20714	-0.61027

Table 1: Summary statistics for the four portfolios.

From Table 1, we can see that the four portfolios were highly peaked and slightly skewed to the left. This was in line with the findings of Clark and Troskie (2006). As expected, the smaller capitalization portfolios had higher mean returns, standard deviation and excess kurtosis than the larger capitalization portfolios.

Distribution Fitting

Fitting Normal distributions to our portfolios' return series simply involved setting the mean and variance parameters equal to the sample average and variance, respectively.

Considering the complexity of the EM algorithm and the small degree of skewness, we opted to fit two-component scale-mixture of Normal distributions where only the variance parameter differed between components. In other words, components shared a common mean but have different scale parameter values.

Name	$weight_1$	$weight_2$	mean	sd_1	sd_2
EWS-portfolio	0.95595	0.04405	0.01565	0.06158	0.25878
EWL-portfolio	0.79400	0.20600	0.00475	0.05656	0.11215
LWS-portfolio	0.93082	0.06918	0.01517	0.05496	0.19233
IWL-portfolio	0.75021	0.24979	0.00482	0.05726	0.11203

Table 2: Estimated parameters from fitting scale mixtures of two Normal distributions.

Scale mixtures were fitted using the GNU R (R Development Core Team 2007) software package and functions provided by the mixtools (Young, Elmore, Hettmansperger, Hunter, Thomas & Xuan 2007) package. The results, Table 2, revealed some intriguing patterns. The high-variance component's weighting was lower for the smaller capitalization portfolios than for the larger capitalization portfolios. Indeed, the high-variance component's weight for the EWS-portfolio and LWS- portfolio was over 4.6 and 3.6 times smaller than that of the EWL-portfolio and IWL-portfolio, respectively. The high-variance component's standard deviation was, however, larger for the smaller capitalization portfolios. Specifically, the EWS-portfolio's and LWS-portfolio's high-variance component's standard deviations were about 2.32 and 1.72 times larger than those of the of EWL-portfolio and IWL-portfolio, respectively. For all four portfolios, the low-variance components' standard deviations were of a similar order of magnitude.

Component	Portfolios	F-statistic	p-value
1	EWS-EWL	1.18539	0.28055
	LWS-IWL	0.92128	0.60338
2	EWS-EWL	5.32430	0.00053
	LWS-IWL	2.94731	0.00819

Table 3: F-test for equality of variances between different portfolio component variances.

Using a standard F-test, Table 3, shows that there was an insignificant difference between the low-variance components of our portfolios. The difference between the high-variance components is, however, highly significant. This was tested further, using a one tail F-test which indicated that the smaller capitalization portfolios' high-variance components possessed significantly larger variances than those of the larger capitalization portfolios' high-variance components.

In terms of a two-state model, the above suggests that the portfolios were exposed to a similar degree of risk when in a low-volatility state. During the high-volatility state, the smaller capitalization portfolios experienced greater volatility, but experienced it less frequently than the larger capitalization portfolios. This is in line with the observation by Kon (1984), when he noted: “The larger frequency of rejection for the E[qually]W[eighted] returns may indicate that smaller firms have fewer but more surprising information releases.”

Goodness of Fit Testing

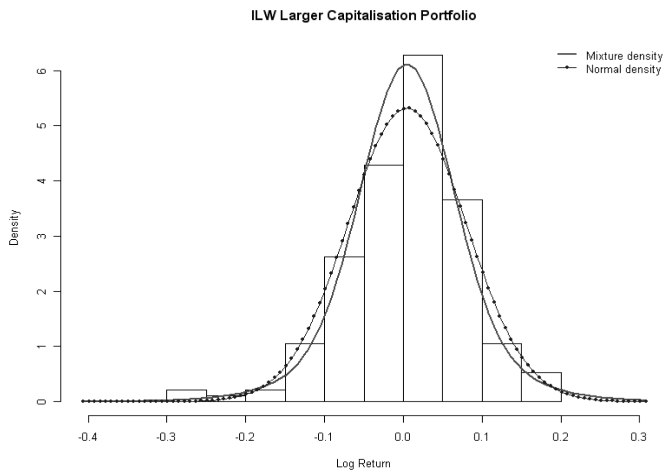

Figure 1: IWL portfolio returns, with fitted Normal and mixture distributions superimposed.

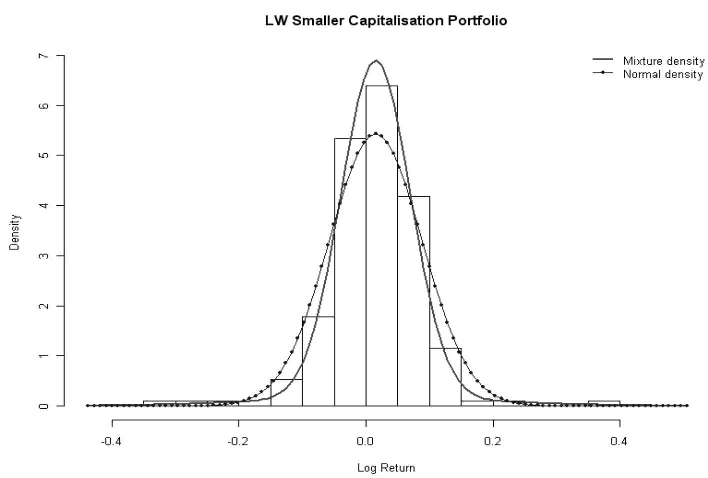

Figure 2: LWS portfolio returns, with fitted Normal and mixture distributions superimposed.

Graphically, Figure 1 and 2, depicts the goodness of fit for the IWL-portfolio and the LWS-portfolio. From these two graphics we clearly can see that the mixture distribution outperforms the Norm distribution. This is especially apparent in the tails of the distribution.

Formal goodness of fit test results, using the Pearson's Chi-squared test, confirm the graphical results. Additionally, note that the mixture distributions performed better for the smaller capitalization portfolios than for the larger capitalization portfolios.

CONCLUSIONS AND RECOMMENDATIONS

This paper showed the clear advantages of using a mixture of Normal distributions as a model of equity returns. The model does not only provide a superior fit, compared to the Normal distribution, but also has an intuitive interpretation in terms of informational or volatility states. Our model also highlighted possible differences in the risks connected to smaller and larger capitalization portfolios. These differences might be due to a differential-information effect, though identifying such an effect is a task for future researchers.

The significant difference between the high-volatility components lends support to the claim that the distributions of the smaller and larger capitalization portfolios' are different. An area of research that can be expanded on is a test for comparing both mixture distributions simultaneously.

Our finding must, however, be interpreted with some caveats. These include the inability to correct for survivorship bias and the exclusion of dividends from the return calculations. Were we able to include dividends and a correction for survivorship bias, we would expect the larger capitalization portfolio to become positively skewed, due to higher dividends, while the smaller capitalization portfolio would most likely become negatively skewed, due to higher levels of attrition. Such return distributions could be modelled using two-component Normal mixtures with arbitrary means and variances.

In summary, scale mixtures provide a flexible model able to capture the differences in the risk structures of size-based portfolios. The nature of these differences, as well as the inclusion of survivorship bias and dividends, provide for future avenues of research.

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