The Costs and Opportunities for Portfolio Diversification in Southern Africa's Smallest Equity Markets

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Abstract

This paper contrasts the forecasting performance of three time series models for three very small frontier equity markets in Africa. In the light of proposed regional equity market integration this study reveals potential benefits from diversification to South African investors from Namibia while Swaziland and Mozambique markets remain segmented. It also indicates the impact of a proposed levy to increase investment retention to retain liquidity and build up local capital markets. Cost premiums would be incurred by domestic investors within smaller markets are found to be persistent across all markets indicating that such a levy should be carefully considered.

JEL classification: G 11, G12, G15, O55

Keywords: Portfolio Choice, Asset pricing, International Financial Markets, Sub-Saharan Africa

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1. Introduction

The drive towards integration of Africa's equity markets has already attracted interest in the literature, see for example, Irving (2005), Yartey and Adjasi (2007) and Piesse and Hearn (2008). This has largely concentrated on the larger markets, many of which are considered future integration hubs that extend outwards from regional centres to smaller surrounding markets. However, there is very little work modelling the smaller "micro" equity markets and those that do focus on single market studies such as Jefferis (1995), Lavelle (2001) and Marone (2003). These markets are not only very small but also complicated by the policy options favoured by governments and regulatory authorities who wish to impose an investment levy to retain liquidity in the home market.

This paper has two primary objectives. The first is to assess the potential opportunities for diversification that a group of very small markets within the Southern African Development Community (SADC) offer to investors based in the largest market, South Africa. The second is to examine the costs incurred by investors within these smaller markets of the policy to require that a minimum percentage of investment funds is retained within the home market. For example, Regulation 28 in Namibia requires a 35% minimum local investment for pension funds (Sherbourne and Stork, 2004).

Three approaches are used to model the time series of the respective markets total returns indices: a simple iid model, which implies a stochastic time series with drift; the generalised autoregressive conditional heteroskedasticity (GARCH) model developed by Bollerslev (1986); and the standard capital asset pricing model (CAPM). When the models have been specified, the one step-ahead forecast of mean and variance-covariance can be used within an optimised portfolio framework to assess the potential costs and benefits of diversification for investors.

There is a considerable literature that uses GARCH and GARCH-in-Mean (GARCH-M) models to capture time series effects within equity price series. Recently, this has centred on the assessment of weak-form price efficiency in the larger African equity markets (Jefferis and Smith, 2005). The present paper follows Zalewska-Mitura et al (1997) and Zalewska and Hall (1999) by applying the Kalman filter to generate time varying parameters of a GARCH-M framework developed with an equation for the mean as a simple AR(1) model. If the hypothesis of weak-form efficiency holds then the coefficient of the first order lagged return should not be significantly different from zero. Jefferis and Smith found that although several of markets became increasingly weak-form efficient over the course of the 1990's, South Africa was the only market where returns were unpredictable for the entire sample period.

The theoretical and empirical literature using CAPM is vast and has recently been enhanced by the addition of covariance risk to other variables commonly used within equity valuation. For example, book-to-market ratios and company size has been incorporated into the original CAPM framework by Fama and French (1993) and Liu (2006) further added a liquidity premium. Liquidity itself is first measured using a pre specified construct, such as that outlined in Amihud (2002), with rankings given to sample group equities, and the premium itself generated through the difference in returns of the highest and lowest ranked assets. Although the liquidity constructs that are common in the literature work well for more liquid assets, Lesmond (2006) notes that these measures are persistently rendered inconclusive for very illiquid assets. Phylaktis and Ravazzolo (2004) explored another common issue in emerging equity market valuation: that of the covariance risk to exchange rate or currency premiums. A CAPM model augmented with a currency premium is found to outperform standard models and is of particular relevance given many emerging markets have undergone recent capital market liberalisations. Finally, Collins and Abrahamson (2006) apply a CAPM type model in calculating the cost of equity for a sample group of markets in Africa, while noting that the standard CAPM type model is common amongst investors in assessing potential opportunities. Because of the difficulties of liquidity measures in very illiquid markets and given all countries in this study are members of a common monetary area, with the exception of Mozambique, currency premiums are unnecessary and the simple one factor CAPM model is adequate.

Modern portfolio theory indicates that investors should hold mean-variance efficient portfolios of assets. However, one recent strand contrasts the ability to forecast means and covariances in terms of the consequent effects on optimal portfolio asset weights. Harvey (1994) compared the predicted mean and covariance matrix from a simple iid model, inferring an unconditional asset allocation strategy where managers have no other information other than historic returns to predict future values to regression models including a variety of world and local market variables as conditioning information. The analysis is focuses on exploring the benefits to investors of diversification into emerging markets through the development of efficient frontiers. Chan et al (1999) assess the forecasting ability of a variety of models both in terms of variance tracking performance to a known benchmark as well as in relative portfolio weights in a minimum variance optimised portfolio setting. The focus is directed towards individual industries within developed OECD markets. In a further paper, Harvey (1995) follows a similar route but concentrates market index level analysis.

This paper is organized as follows. Section 2 has two distinct parts: the first provides an overview of the institutional features of the four sample markets, while the second discusses data specific issues. Section 3 outlines the three approaches to modelling the total returns series, iid, GARCH and CAPM, as well as describing the application of mean-variance portfolio optimisation techniques. Section 4 discusses the empirical results. The final section concludes and provides development policy inferences based on the evidence from this work.

2. African Equity Markets

i) Institutional Characteristics of Emerging Financial Markets

Four emerging markets are examined in this paper and there are clear differences in their institutional design, market capitalisation and level of development. The major characteristics of these markets are summarised in the following sections (see Piesse and Hearn (2005) for an extended discussion of African stock markets):

South Africa. The Johannesburg Stock Exchange (JSE) is the largest market in Africa. It is the most developed and is well regulated. The JSE adopted the Stock Exchange Electronic Trading Shares, or SETS, order-driven electronic trading platform used by the London Stock Exchange in 2002. Trading takes place daily and the market has a pre-opening electronic call auction 8-25am and 9-00am and continuous trading 9-00am to 4-00pm. Despite being classified as an emerging market there is high institutional investor participation and ownership is high diversified (Bloomberg LP, 2006). Settlement is through a central depository on a rolling contractual basis of trade date plus five working days (T + 5) and is largely G30 compliant (STRATE website, 2007).

The South African market has experienced two distinct periods of transition during the period of this study. The first was 1990 to 1995 when the market was closed to foreign investors, largely due to sanctions by the rest of the world. Also at this time domestic investors had to comply with the National Party's "prescribed assets" regulation, which emphasised investment in domestic equities rather than money or bond market instruments (Grandes and Pinaud, 2004). The second follows the ending of apartheid in 1995 and the subsequent real and financial market liberalisation that followed, including the opening up of markets to foreign institutional investment, the move to electronic trading and the introduction of formal legislation to ensure international levels of corporate governance.[1] Further revision of the Kingly report in early 2000 has led to increased investor confidence and market development although competitiveness has been hindered by volatility of the domestic currency and high risk premiums that have a negative impact on overseas investors

[1] The Kingly Report that regulates corporate governance practices in South Africa is very similar to the UK Cadbury Report and the US Sarbanes Oxley Act.

(Grandes and Pinaud, 2004). This has also resulted in a loss of liquidity in the domestic market and the tendency for primary listings to take place on overseas exchanges such as London and New York in preference to the JSE.

Namibia. The Namibian Stock Exchange (NSX) is the second largest market in the Common Monetary Area (CMA), or Rand Zone. It shares the electronic trading system, central depository, market infrastructure, settlement cycle and reporting with South Africa and given these common factors the markets have a high degree of integration (Hearn and Piesse, 2002). However, despite the advanced stage of development, the market lacks liquidity and some 68% of 31 current listed companies have primary listings in Johannesburg. There is also evidence of further decreases in liquidity levels in the local primary listed market, which is partly due to dual listed entities being classified as Namibian. Block trades in these assets are booked in Johannesburg with only surplus unmet demand being carried across to the related Namibian shares. This provides evidence that Regulation 28, which is the formal policy to ensure that a minimum 35% of funds are invested in domestic assets, is not working (Sherbourne and Stork, 2004).

Swaziland. The Swaziland stock exchange in Mbabane was incorporated in July 1990 and since inception has been hindered by both lack of infrastructure and formal regulation. Trading was initially conducted by a single broker until a second brokerage house was established as an over-the-counter in 1998. The Securities Markets Regulation Bill has still to be ratified by parliament and not surprisingly, this has been a major barrier to the growth and development of the market. However, in 2000 the Central Bank of Swaziland extended the existing Financial Services regulation to cover the stock market, and has also provided some trading facilities within the Bank premises. To minimise costs, Central Bank staff provide both trading and building maintenance duties. Trading takes place daily 10-00am and 12-00 midday as a call auction where orders are pooled and then matched and executed at the price that most accurately reflects all information available to market at time of trade.

Interesting, although liquidity is very low, Swaziland is part of an economic and currency union with South Africa and therefore domestic currency premiums are directly comparable to others in the rand zone. However, given the small size of market and lack of regulation only one company, Masterfridge, during the course of its history was dual listed in nearby Johannesburg. Owing to lack of domestic demand the local brokerage community relied on the South African market for the price discovery mechanism and price quotes were given in line with those prevailing in Johannesburg to prevent potential cross border arbitrage transactions (Matimela, 2007). Despite the size of this market two brokerage houses, African Alliance and South Africa's Interneuron Asset Manager, have registered as local dealers since 1998. Asset allocations in favour of Swazi listed instruments in excess of those in South Africa and other regional centres is very low, often less than 5% of portfolio inventories

(African Alliance, 2007). Ownership is highly concentrated and free float capitalisations available to the public are low (Swaziland Stock Exchange, 2007). Settlement is by physical transfer of assets between the holdings of local banks with sufficient capitalisation to act as market custodians.

Mozambique. The Maputo Stock Exchange, the Bolsa de Valores de Maputo (BVM), is a fledgling market with only one listed stock, the former national brewing company Cervejas de Mocambique. This listing followed a joint venture with South African Breweries, which was part of the privatisation programme recommended by the international financial institutions and development agencies. Despite its small size this market has been well designed from inception, with assistance from the New York and Lisbon exchanges. However, regulation is weak, particularly with respect to the protection of small and minority investors. Trading is predominantly government treasury bills and bonds with maturities of up to 3 years, as the exchange acts as a second outlet debt issues outside the Central Bank auctions. Despite a network of licensed brokers including recently privatised commercial banks, trading activity in the single equity remains tightly focussed on providing the interbank market for debt with an additional instrument to broaden holding portfolios.

Trading is by a delocalised electronic order book system where trades are matched on a price-bargain basis. Licensed brokers have access to an online trade entry system run from the Stock Exchange website. Levels of illiquidity are severe with trades taking place only once every several months and the time from submission of an order to market execution can be 25 working days. Official trading hours are 10-00am to 12-00 midday on Tuesday, Thursday and Friday (BVM, 2007). Free float percentages are some of the lowest in entire continent and have fallen since listing in 2001 from 6-5% to the current level of 4% (BVM, 2007). Ownership is highly concentrated and settlement by local market custodians similar to Swaziland. Trading and settlement activity for the single listed equity is dominated by the South African broker Standard Bank. This is a clear indication of the lack of investor confidence in local institutions, which is not surprising given the 1994 financial crisis when two newly formed commercial banks failed due to bad debt overhang from the previous socialist era.

All four markets have poor liquidity compared with developed world markets, but particularly Swaziland and Mozambique, which are two of the smallest in Africa. They both lack transparency, liquidity and fall far short of best practice in terms of corporate governance. In Swaziland there is a complex state ownership structure of cross holdings between various listed entities. In Mozambique the very small formal sector has only recently adopted auditing techniques into national accounting practices (Standard Bank, 2007). They are both a very long way from the highly regulated standards in South Africa and Namibia. Given these markets very small size, varying degrees of integration, risk and liquidity, together with their common membership of the CMA and consequent lack of exchange rate risk they provide a unique opportunity to assess the performance of time series models, within a mean-variance optimised portfolio framework.

ii) Data: Sources and Total Returns Indices

Values of the end of month total returns of the market index for South Africa are from Datastream. The Standard & Poors frontier market total returns index was used for Namibia from December 1998, when the market was established. For Swaziland and Mozambique, total returns indices for stocks are not available through local or international media sources, and so were collected directly. Stock splits and rights issues, stock prices, number of shares outstanding, dividend payments history, daily trading volumes and market capitalisation data were from the national Stock Exchanges. Individual stocks total returns indices were constructed and then aggregated to form a market index following the method used by Standard & Poors. Companies that are delisted were not deleted from the sample prior to their delisting to prevent survivorship bias.

Prices in Namibia and Swaziland are quoted in local currency, but transformed to South African Rand. As Mozambique is outside the CMA the data series were converted to Rand in order to present the South African and CMA investor perspective. This also removes the effects of high and volatile local currency premiums from the calculation of excess returns. The exchange rate data is from Datastream and the central bank. The one-month South African Treasury Bill yield rate represents the risk free rate although this is adjusted to take account of monthly excess returns as opposed to the quoted equivalent annualised rates. The conversion of the total returns series and prices by South African Rand exchange rates rate assumes long term parity between individual domestic currencies and the US$. The use of the South African Treasury yield within the context of all time series denominated in South African Rand is entirely appropriate as this reflects analyst best short term expectations of economic conditions within CMA and South Africa's economy. South African Treasury Bill data are from Datastream. The use of the South African Treasury Bill as a risk free rate is not ideal given the high and variable risk premiums associated with this rate (Grandes and Pinaud, 2004). However it is the most stable Treasury rate in a region with a history of high and volatile inflation and all CMA member countries (Namibia and Swaziland) have interest rate movements tied very closely to those in South Africa by nature of the currency board. It would also be reasonable to assume that the smaller less capitalized and largely segmented investor community in the sample group markets would be unlikely to be able to access conventional Treasury instruments, such as those of US and Europe, in order to include these within local portfolios.

3 Models

This section first considers the two classes of model employed, namely the unconditional investment strategy implied by the naive iid model, and the two conditional strategies, GARCH and CAPM that use time series autoregressive and market risk premium conditional components respectively. Then, portfolio construction is reviewed and optimisation techniques discussed within a minimum variance framework.

i) IID model: Unconditional asset allocation

This strategy is considered unconditional as it implies that there is no other information relevant for forecasting the next period's price other than the previous price, that is, stock returns are not predictable. The expected returns are modelled as a rolling window of the mean returns over the previous 24 months. Despite the movement in these mean returns as the 24-month window moves, using the average returns assumes that the best forecast of the equity returns is its past average. This is consistent with the random walk model of stock prices with a drift component and has a core assumption that the underlying time series is weak form efficient.

The 24 month rolling window is extended for the calculation of standard deviation, and hence variance, as well as correlations from which updated covariance estimates are calculated. The simple myopic model can be represented by

$$y_t = \mu + \varepsilon_t \ldots\ldots \varepsilon_t \sim N(0, \Sigma), \tag{1}$$

in which μ and Σ are matrices of constant parameters. Recursive rolling window estimation is necessary for μ and Σ in order to generate vectors of sample means and covariance respectively.

ii) GARCH model: Conditional asset allocation

As discussed in Bollerslev (1987), there is evidence indicating that the change in prices and rates of return are approximately uncorrelated over time, but characterized by tranquil and volatile periods. Allowing for such dependence this study takes the conditional mean $y_{\langle t|t-1\rangle}$ as being dependent only on its first order lagged counterpart and a constant,

$$y_t = \alpha_0 + y_{t-1} + \varepsilon_t \tag{2}$$

together with a GARCH (p, q) model for the conditional variance:

$$E\left(\varepsilon_t^2 \mid \psi_{t-1}\right) = h_{t|t-1} \tag{3}$$

$$h_t = \omega_0 + \sum_{i=1}^{p} \delta_i h_{t-i|t-1-i} + \sum_{j=1}^{q} \gamma_j \varepsilon_{t-j}^2 \tag{4}$$

where $\omega > 0$, $\delta \geq 0$, $\gamma \geq 0$. It can be seen that in the model given by (2) and (4) that there is a tendency for large (small) residuals to be followed by other large (small) residuals but of unpredictable sign. Equation (4) effectively says that the value of h_t depends on past values of shocks (the *q* moving average terms), which are captured by the lagged squared residual terms, and on past values of itself (the *p* autoregressive terms), which themselves are captured by lagged h_t terms. Furthermore the work undertaken in Bollerslev (1987) allows for the relaxation of the assumptions concerning mildly leptokurtic conditionally normal errors, and the adherence to conditionally t-distributed errors. This gives the GARCH model greater flexibility with financial time series exhibiting very fat-tailed distributions, which are typically exhibited in emerging market time series.

iii) CAPM model: Conditional asset allocation

The simple Capital Asset Pricing Model, CAPM is used in this paper. This model assumes market equilibrium under conditions of risk (Sharpe, 1964) and takes account of options faced by investors and optimal valuation of assets (Lintner, 1965). These models make a number of critical assumptions, which may be invalidated by the data time series considered within this study, although due to the lack of alternatives are still in use on trading floors (Collins and Abrahamson, 2006). The most controversial assumptions within the mean-variance framework are those alluding to the weak-form efficiency of all component time series, and investors utility functions may be modelled by functions with moments no higher than two. The latter assumption has been the source of recent critical investigation by Canela and Collazo (2007), who provides evidence of higher order utility functions in emerging markets. In contrast to these findings, Lintner's original formulation asserted that the quadratic function implies negative marginal utilities of income or wealth much "too soon" in empirical work unless the risk aversion parameter is very small, in which case it cannot account for the degree of risk aversion found. It also implies that over a major part of the data range common stocks are regarded as "inferior goods". Thus, offering more return at the same risk would so sate investors that they would reduce their risk-investments because they were more attractive. Circumventing this issue, this paper provides a simple analysis within the expected mean – variance linear framework following Mossin (1969) and Cheng and Grauer (1980). Much of this framework is commonly reported in financial and econometric textbooks.

Following the initial derivation of CAPM as outlined in Cheng and Grauer (1980) is the assertion that for a given period t in a mean-variance optimizing world the model states that if investors have homogeneous beliefs, where μ_{jt} is the vector of mean returns at t, and

$\sum_t$, is the variance-covariance matrix then the mean return of an asset or portfolio j is linear with respect to its systematic risk, β_{jt} . This is the securities market line, SML,

$$\mu_{jt} = \bar{r}_{zt} + \beta_{jt}(\bar{r}_{mt} - \bar{r}_{zt}) = \bar{r}_{zt}(1-\beta_{jt}) + \bar{r}_{mt}\,\beta_{jt} \quad (5)$$

where $\bar{r}_{zt}$ represents the risk free rate of return in time t. $\bar{r}_{mt}$ is the mean return of the market portfolio in time t, and β_{jt} is the systematic risk of security j at time t. Cheng and Grauer (1980) note that μ_{jt} and $\sum_t$ are exogenous while $\bar{r}_{zt}$, $\bar{r}_{mt}$ and β_{jt} are endogenous and the SML designated in (5) is an equilibrium relationship not a function. Furthermore, supposing that (5) holds for each t, $t \in T$, where t is entire sampling period, then the return generating process in Sharpe (1964) and Lintner (1965) is that of a stochastic stationary process,

$$\tilde{r}_{jt} = \tilde{r}_{zt}(1-\beta_j) + \tilde{r}_{mt}\,\beta_j + \tilde{e}_{jt}, t \in T \quad (6)$$

where the tildas denote random variables that follow stationary stochastic processes and both the mean of $\tilde{e}_{jt}$ and its covariance matrix with other random variables are assumed to be zero. It is important to note that the beta is no longer time scripted, implying a constant value across all t within the entire sample period of T. The imposition of a fixed β_j in the traditional SML tests of CAPM carries with it the underlying assumptions of return distribution stationarity as well as the assumption of a constant $\bar{r}_{zt}$.

However, the CAPM is a linear function that imposes strict assumptions on the structure and composition of the error matrix, as discussed above. But these assumptions of zero mean and non-diagonal elements of variance-covariance matrix in relation to other time series of the $\tilde{e}_{jt}$ term in equation (6) are unrealistic given the often severe deviations from normality experienced in emerging markets and especially in smaller frontier markets such as those within this sample. There is likely to be very high and persistent levels of autocorrelation and heteroskedasticity within the error and variance terms in a linear or multivariate regression model that would significantly bias the estimators. Thus, this study uses the CAPM to account for the mean returns in all cases but in addition, represents the variance and error matrix within a GARCH formation. The optimal lag order is determined by the Akaike information criterion as the R^2 terms are not useful due to the very different construction of estimators in GARCH type models. Thus, the following is the mean equation in a CAPM formation,

$$\tilde{r}_{jt} = \tilde{r}_{zt} + \beta_j (\tilde{r}_{mt} - \tilde{r}_{zt}) \tag{6a}$$

together with a GARCH (p, q) model for the conditional variance:

$$E\left(\varepsilon_t^2 \mid \psi_{t-1}\right) = h_{t|t-1} \tag{6b}$$

$$h_t = \omega_0 + \sum_{i=1}^{p} \delta_i h_{t-i|t-1-i} + \sum_{j=1}^{q} \gamma_j \varepsilon_{t-j}^2 \tag{6c}$$

where $\omega > 0$, $\delta \geq 0$, $\gamma \geq 0$.

iv) Portfolio Strategies: Unconditional versus Conditional

Following Jackson and Staunton (2003) and Harvey (1994) within the mean-variance framework originally established by Markowitz (1959) an investor's utility preference function can be expressed as follows

$$U = E(r_p) = \frac{1}{2}\lambda\sigma_p^2 \tag{7}$$

where the higher the value of the risk aversion coefficient, λ, the larger the portfolio risk that is subtracted from the portfolios expected return. At a given λ the portfolio's return and variance can be expressed as follows:

Portfolio Return (conventional notation): $E(r_p) = \sum w_i E(r_i)$ (8a)

Portfolio Return (matrix notation): $w^T \mu$ (8b)

Portfolio Variance (conventional notation): $Var(r_p) = \sigma_p^2 = \sum\sum w_i w_j \operatorname{cov}(i, j)$
where $\operatorname{cov}(i, j) = \sigma_i^2$ (9a)

Portfolio Variance (matrix notation): $w^T V w$ (9b)

where $E(r_i)$ denotes the expected return for the i[th] asset, and σ_i is the risk (standard deviation of returns) for the i[th] asset. w represents N x 1 vectors of weights of individual assets and V is the N x N variance-covariance matrix, and N is the number of markets under consideration, where N = 2 in this paper.

Unconditional Strategy

In this strategy the investment proportions are unrestricted in size but must sum to unity. However, this does allow extremely large short and long positions in any of the markets under consideration. As thin trading is a widespread problem across emerging markets, and particularly so in the three non-South African markets, a plausible constraint should be that all short sales be disallowed, that is, $w_i \geq 0$ for i = 1,….N. This adds a second constraint to the mean-variance optimisation problem.

The strategies evaluated in this paper involve solving (8a), subject to the global minimum of (9a) at the end of each month and holding the implied portfolio for the next month. The sample is updated using a 24-month moving window and the portfolio is re-optimized at each point in time. In all strategies, transactions costs are ignored. In each case the minimum variance portfolio is analysed, that is, the investment weights match the weights implied by the minimum variance portfolio over the previous 24 months. These weights are used to form a portfolio that is held over the next month.

The strategy is considered unconditional because of the way that expected returns, variances and covariances are selected. The expected returns are the rolling mean returns over the previous 24 month window. Although these mean returns change through time as the 24 month rolling window moves, using the average returns assumes that the best forecast of the equity return is its past average. This is consistent with a random walk model of stock prices with drift, as defined above.

The unconditional strategy also places restrictions on other inputs to the optimisation problem. The variances and covariances are assumed to be unconditional variances and covariances over the previous 24 months. This precludes the possibility that these measures move in more complex ways. Although the optimisation function is considerably more sensitive to changes in the expected return, the recursively updated variance-covariance matrix values are also of importance.

All strategies are in South African Rand and assumes that no currency hedging takes place, which is reasonable given the CMA. In the specific case of Mozambique, if this problem were to be implemented in local currency terms this would be consistent with perfect foresight hedges being initiated relative to South Africa. While this assumption is tenuous for developed and larger emerging markets it is unreasonable for smaller emerging markets and assumes the availability of suitable futures and forward contracts as well as analyst coverage generating a sufficient supply of economic information. As a result the evaluation is undertaken in a common numeraire currency[2].

Conditional Strategy

In the mean variance optimisation problem in (8a)/(9a), three sets of inputs are needed: means, variances and covariances. At the end of each month the investor attempts to design the portfolio that guarantees the highest possible expected return for the minimum level of volatility. In solving the unconditional problem a set of portfolio weights are obtained that guarantees the highest ex post returns for minimised level of volatility over the past 24 months. Thus, the unconditional mean-variance problem delivers a set of investment weights that are relevant only over the past history. The only way for the manager to obtain an efficient portfolio in the unconditional problem is to hold the investment weights implied by the actual data, but evaluation is assessed on the basis of the efficiency of managers derived from their ability to construct a portfolio as close as possible to the efficient frontier. However, using the unconditional strategy the only way the manager can obtain this frontier portfolio is by knowing the historic data which guarantees the portfolios efficiency over the past but not in the future.

Using conditional models is preferential for the mean-variance problems in terms of opening the possibility to provide the best possible forecasts of the expected returns, variances and covariances for the next period. The past averages are not so meaningful due to the investment manager focussing more on the future and not the past and as such past averages are only of benefit when the means, variances and covariances are completely unpredictable. This highlights the rationale for inclusion of the unconditional model within the portfolio optimisation problem for this sample, given the cited difficulties in the literature of conditional model construction for very small micro-emerging markets where extremely high illiquidity renders many time series models obsolete in capturing effects in the time series.

The conditional asset allocation implements forecasting models for the inputs of the mean-variance problems. Linear regression models are constructed for the conditional means in the case of CAPM and a first order auto-regression is undertaken for the mean equation of

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[2] Harvey (1993b) analyses the international asset allocation problem and makes the assertion that the portfolio selection should include currency portfolios, in the form of local deposits or loans. The solution to the quadratic program should deliver the optimal asset allocation as well as optimal currency hedges. It is likely that this method would be preferential to models where perfect foresight of exchange rates is required, such as currency covariance augmented CAPM models as holdings of existing domestic deposits could be recursively modified within the optimisation process.

the GARCH model. The regression models of conditional means use a number of information variables,

$$E(r_{it} \mid Z_{t-1}) = Z_{t-1}\delta_i \quad (10)$$

where r_{it} is the return on country *i* from t–1 to T and Z_{t-1} is a 1 x *l* vector of *l* global or market specific information variables. In the CAPM it is the market premium, while in GARCH it is the first order lagged dependent variable, known at time t–1, and δ_i is a *l* x 1 coefficient matrix. The errors from these mean regressions, ε_{it}, are assumed to be unrelated to the conditioning information Z_{t-1}.

As linear models are used, (see Solnik, 1993 for full evaluation) the forecasting variables include a constant in the case of GARCH although this is omitted in the CAPM construction owing to the constant being indifferent from zero by construction. The market premium used in the CAPM is a financial time series index, which itself is available on the last day of the month ensuring a suitable time frame of data availability.

The portfolio problem also requires the forecast of the variance-covariance matrix. Consider the covariance between asset i and j:

$$\text{cov}[r_{it}, r_{jt} \mid Z_{t-1}] = E[(r_{it} - E[r_{it} \mid Z_{t-1}])(r_{jt} - E[r_{jt} \mid Z_{t-1}]) \mid Z_{t-1}] \quad (11)$$

Given the regression errors in (10) it is possible to rewrite (11) as,

$$\text{cov}[r_{it}, r_{jt} \mid Z_{t-1}] = E[\varepsilon_{it}\varepsilon_{jt} \mid Z_{t-1}] \quad (12)$$

The conditional covariance is the forecasted value of the product of the residuals for the regression models for asset i and asset j.

In principal, the conditioning information for asset i and asset j is different. Additionally the conditioning information on the product of the two residuals could be the intersection of the two information sets plus additional variables. Therefore, the GARCH model includes lagged values of the product of the residuals in the information set.

The approach used in this paper follows Harvey (1994) by using the unconditional mean of the product of residuals as the forecasted variance-covariance matrix. This implicitly assumes that the product of the residuals is not predictable and follows Solnik (1993). However, the matrix used in this paper is not the unconditional variance-covariance matrix but the average conditional variance-covariance matrix where the Z_{t-1} variables are permitted to affect the means. This approach greatly simplifies the estimation.

In a similar manner to the unconditional asset allocation, the variance-covariance matrix is based on a 24 month moving window average of the product of the regression residuals through the sample. In the analysis the regressions are estimated over the full sample which implies that the regression coefficients, δ_i, are constant.

4 Results

i) Summary Statistics

Summary statistics for all four component markets are presented individually in Tables 1 and 2 for the entire period. In all cases the total returns indices are reported in Rand and explicitly take into account full reinvestment of dividends as well as stock splits and other corporate actions. Table 1 reports the annualised arithmetic and geometric mean, standard deviation and autocorrelations.

Table 1

The arithmetic mean returns for the sample group exhibit a large variation with Namibia the lowest (1.07%), and South Africa (1.59%) and much higher values of Swaziland (1.76%) and Mozambique (2.33%). There is a similar profile for the standard deviation with Namibia (6.34%) and South Africa (5.94%) and much higher values for Swaziland (9.82%) and Mozambique (14.75%). The autocorrelation profiles of the four markets demonstrate that South Africa has no autocorrelation coefficients over 10% for the first 24 lags, which is in line with results for tests of weak-form price efficiency reported in Jefferis and Smith (2005). In contrast, Namibia and Swaziland have only one lag with an autocorrelation coefficient of greater than 10%. In the case of Mozambique, there are three coefficients of the lagged autocorrelation that are over 10% and the first order lag has a value of 18.8%, which suggests that returns in this market are predictable given past information. In contrast to previous studies analysing the price indices of Namibia and South Africa (Piesse and Hearn, 2002, and 2005) where consistently high values of correlation are found there is no evidence reflected in the relationship between total returns indices. All correlations reported in Table 1 are low indicating a general lack of correlation between these markets, which in turn implies considerable potential for diversification opportunities for local investment fund managers.

The geometric means in Table 1 are important as these reflect the average returns to a buy and hold strategy, which is typically employed by institutional investors and pension funds and is especially evident when a minimum domestic markets investment retention policy is in force. The presence of very high volatility can cause significant differences to emerge between the arithmetic and geometric mean returns. This is very clear in these countries, especially in Mozambique where the arithmetic mean is 2.33% and geometric mean is 0.89%.

Further descriptive statistics are reported in Table 2. These highlight the large difference between the arithmetic mean and median values particularly in the series for Swaziland and Mozambique and to a much lesser extent for Namibia and South Africa. Swaziland and Mozambique have the greatest differences between maximum and minimum. Swaziland consistently exhibits the greatest degree of skewness (1.1809) and kurtosis (9.8521) while Mozambique demonstrates very little skewness (0.4339) and kurtosis

(4.3031). Given the extreme differences of liquidity across the sample it is quite likely that this explains the very high values of skewness and kurtosis in Swaziland, whereas lowest values in Mozambique are likely caused by equally extreme inactivity within this market with trades taking up to 24 days to execute (Standard Bank Maputo, 2007).

Table 2 insert here

Jarque-Bera tests for normality in Table 2 provide strong evidence of non-normality of returns in South Africa and Swaziland, while Namibia has a high degree of normality. The results for Mozambique are ambiguous, given the high level of autocorrelation in Table 1, and extreme illiquidity. Despite the above differences in diagnostic statistics for the sample the hypothesis of a unit root, or stochastic martingale trend, is clearly rejected for all returns series.

ii) Performance of unconditional asset allocation strategy

The unconditional minimum variance frontiers are shown in Figures 1(a) to 1(c) and the recursively optimised portfolio holding weights are in panel 1 of Table 5 for each two asset portfolio universe of South Africa combined with Swaziland, Namibia and Mozambique respectively. Figure 1(a) demonstrates that using this strategy there is some potential for diversification in including Swaziland alongside South Africa, although the investment holding of South Africa dominates, indicating that optimal portfolios are formed with South Africa taking a major holding. Given greater integration between South Africa and Namibia, Figure 1(b) shows a more even investment weighting between the two markets. The asset allocation profile for the portfolio including Mozambique shows a clear trend throughout the duration of sample period with the Mozambique weight decreasing towards 0% with South Africa, correspondingly increasing towards 100%.

Figure 1

In Figure 1, there is considerable variation in the size, shape and duration of the frontiers between the three different portfolios. Figure 1(a) shows the frontier for South Africa and Swaziland, which appears unexpectedly flat and vertical throughout the period indicating that there are significant investment opportunities to be gained through changing asset allocation weights in the portfolio. The flat vertical frontier delineates the possibilities in terms of changing the asset weights to improve the overall portfolio return in relation to an insignificant change in the risk. As expected the frontier of the Namibia – South Africa portfolio in Figure 1(b) is consistently flatter than that in that containing Swaziland but also the frontier is further forward reflecting even less risk, given these two markets are highly integrated. The profile of the Mozambique – South Africa portfolio, in Figure 1(c) is in complete contrast to the other two where, with the exception of one or two points, the entire frontier is close to horizontal. This indicates that varying asset weights results in a very high change in risk in proportion to any incremental changes in overall portfolio return.

The results in Figure 1 and panel 1 of Table 5 are borne out in Tables 6 and 7 which report the performance of the portfolio strategies and the relative cost premiums in enforcing a minimum 30% holding, or asset weight, in the non-South African markets in each portfolio. Table 5 shows that the Mozambique – South Africa portfolio has both the highest annualised return and the second highest annualised standard deviation across the sample for the unconditional strategy. The returns profile also exhibits the largest difference between minimum and maximum annualised returns values. The Namibia – South Africa portfolio demonstrates the lowest return (15.24%) but also the lowest risk (66.67%) with both measures falling short of those for the Swaziland – South Africa portfolio, which has an 18.94% return and 82.11% risk level.

These results are further reflected in Table 7 where yearly annualised geometric means are presented in basis points of the returns differences between a 0% and 30% minimum holdings in the non-South African market. These clearly indicate sizeable premiums likely to be incurred by investors forced to carry a minimum 30% asset weight in the smaller markets, especially Swaziland and Mozambique. There is a very small premium for having a 30% minimum weight in Namibia.

iii) Performance of conditional asset allocation strategy

Tables 3 and 4 outline the coefficients for both the mean equations in each model as well as the coefficient values for all lags within GARCH construction for error matrix. The total returns index used as a proxy for the market return in the CAPM model is the Standard & Poors Emerging Middle East and Africa index while the treasury yield used in implicit calculations of excess returns is the South African 3 month Treasury Bill rate converted into monthly values. Thus, the **β** in CAPM measures the beta to the market while **β** in the GARCH only model is the coefficient on the first order lagged autoregressive variable.

Tables 3, 4 and 5

In all cases in Table 4 within the CAPM models the **β** against the market premium was found to be significant, although especially so for South Africa in each case and then progressively to a lesser extent in the markets of Mozambique, followed by Namibia and finally Swaziland. Namibia was the only series found to have a negative **β** or relation to the market premium. This would infer that Namibia is a strong hedge against the market portfolio, though is more likely to be caused by serious distortions in the Namibia data due to the effects of illiquidity and small listed company size. The inclusion of variables that have coefficient confidence levels of as low as 70% within the mean equation for each series is merited by the necessity of including at least some relation to the market and necessary flexibility in confidence testing within the deductive hypothesis testing empirical framework owing to the difficulties of fitting standard models to such small markets data. The parameters in the GARCH only models in Table 3, that have a simple first order autoregressive term and constant in the mean equation, also demonstrate similar issues to the CAPM-GARCH model coefficients in relation to confidence levels acceptable in fitting models with the data. However, in each case, apart from the Namibia GARCH (1,1), South Africa GARCH (2,6) and South Africa GARCH (3,1) models, the autoregressive term is significantly different from zero at a confidence level of greater than 60%.

Figures 3 and 4

Figure 4 and panel 2 of Table 5 show asset weight allocations between South Africa and each of the other three markets within the two asset portfolio universe. In each case a similar profile, though accentuated in the results using the CAPM model, can be seen to that of the unconditional model. Panels 2 and 3 of Table 5 show that there is a persistent decline in the weight of Mozambique in its portfolio, while holdings are relatively evenly distributed between Namibia and South Africa and the holding of South Africa dominates in the portfolio containing Swaziland. These results are also confirmed in the loci of the investment frontiers (Figures 2 and 3). The frontiers of the Mozambique portfolio (Figures 2(c) and 3(c)) show similar but much stronger profiles to that reported for the unconditional asset allocation strategy. The frontiers are virtually horizontal in the case of the CAPM model while being heavily skewed towards horizontal for the GARCH only model. The frontiers for portfolios containing Swaziland (Figures 2(a) and 3(a)), and Namibia (Figures 2(b) and 3(b)), exhibit better characteristics in the CAPM than the GARCH. The Namibian portfolio frontiers in the CAPM show a more vertical profile implying more investment opportunities with little variation of risk relative to the GARCH only construction. In contrast, the GARCH model has a frontier that is squashed together and shows a narrow range of potential portfolio returns while sitting far back from the standard deviation axis implying that the few opportunities that do exist all carry high risk. Portfolios containing Swaziland in both cases have similar profiles, with the frontiers having the same level of risk and a similar dispersion of vertical returns.

Table 6 provides evidence that the performance of the GARCH only model is inferior to that of the CAPM-GARCH construction. The values of annualised mean return and standard deviation for all three two-asset portfolios are similar to that in the unconditional case. This is characterised by high mean returns and equally high standard deviation values. The CAPM type construction, with GARCH error representation, provides higher values of mean and lower standard deviation despite greater differences in each case between maximum and minimum values of annualised returns. This is further borne out in Table 7 where the portfolios containing Swaziland and Mozambique using the GARCH only model exhibit relatively huge and persistent basis point premiums from an enforced 30% holding constraint as compared to portfolios containing Namibia. In contrast, the retention cost premiums derived from using the CAPM type construction are apparent for all three portfolios yet while showing a similar profile of large premiums incurred in enforced holdings of Swaziland and Mozambique in relation to a holding of Namibia, the values are much smaller in contrast to the GARCH model. The results demonstrate that across all three models, both unconditional and conditional, there are persistent and high premiums incurred from a minimum 30% holding in the very small markets of Swaziland and Mozambique. The premiums incurred from a minimum holding of Mozambique are considerably larger than those of Swaziland inferring that this market is considerably segmented from South Africa.

Tables 6 and 7

5 Conclusions

This study reviews and assesses the potential for portfolio investment between South Africa and the much smaller neighbouring markets of Swaziland, Namibia and Mozambique. The sample is broadly split into two separate groups: those of South Africa and Namibia and separately the two extremely small markets of Swaziland and Mozambique. This division is clearly seen with South Africa and Namibia having comparable mean returns, standard deviation and autocorrelation profiles compared to the considerably higher values for Swaziland and Mozambique. In contrast to previous work that has found substantial correlation between the price indices of South Africa and Namibia the cross correlations between all markets are found to be minimal.

Three models are applied to the time series with considerable differences in relation to the applicability and benefits of the use of each. The practical applicability of the unconditional iid strategy is questioned as it is only necessity for an investment manager to only be able to predict future trends on the basis of past activity, despite the intuitive appeal of this strategy and the logical results obtained. This model shows that South Africa dominates the asset weights in it's portfolio with Swaziland, while the weight of Mozambique is steadily decreased almost to 0% while Namibia holding is roughly equal for the respective time periods. In terms of the two conditional strategies it is found that the CAPM with GARCH representation of errors outperforms the GARCH only strategy.

The portfolio frontiers generated are broadly similar in all cases across unconditional and conditional models, with their profiles being accentuated in the conditional cases. The frontiers, representing investment opportunities in terms of incremental changes in portfolio mean return to proportional amount of risk, are almost horizontal in all cases for the South Africa – Mozambique portfolios. This implies that a portfolio including Mozambique carries a disproportionately high level of risk with respect to any changes of asset weights of the two component assets. Frontiers for South Africa – Swaziland are flatter and more vertical implying a much greater level of potential investment opportunities in relation to relatively small proportional increases in risk. The flattest and most vertical profiles of all are those of Namibia within a South African portfolio. This is intuitively expected given the highly integrated nature of these two markets.

All portfolio strategies, unconditional and conditional, provide evidence of significant cost premiums that would likely be incurred by domestic investors if the smaller markets were forced to adopt a 30% minimum local asset investment retention holding for their portfolios. All models support a very large cost premium that would be incurred with a minimum holding of Mozambique and to a lesser extent with Swaziland. The results in both cases show that the GARCH models report very large premiums despite this model being a poor fit, and the CAPM-GARCH model reporting small but persistent costs. The minimum inclusion of Namibia is every case carries little cost premium compared to the other two micro markets.

Overall this study provides substantial evidence that there is considerable scope for diversification of investment portfolios between South Africa and Namibia with very little benefits relative to proportionally increased levels of risk for Swaziland and Mozambique. However, the costs borne by local investors in these markets in relation to the adoption of a minimum local investment policy of 30% in order to boost small domestic exchanges and attempt to stem liquidity flow would be considerable.

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Table 1. Means, Standard Deviations and autocorrelations of market total returns indices

Country	Start	Arithmetic Mean	Geometric Mean	Standard Deviation	Autocorrelation P_1	P_2	P_3	P_4	P_6	P_{12}	P_{24}
South Africa	1990M7	0.0159	0.0140	0.0594	0.003	-0.019	-0.040	-0.074	-0.004	0.051	-0.002
					(0.967)	(0.963)	(0.938)	(0.814)	(0.454)	(0.732)	(0.990)
Swaziland	1990M7	0.0176	0.0131	0.0982	0.049	0.061	-0.067	0.019	0.114	-0.011	0.092
					(0.474)	(0.525)	(0.525)	(0.679)	(0.182)	(0.452)	(0.191)
Namibia	1999M1	0.0107	0.0077	0.06342	-0.006	0.065	0.124	0.068	-0.004	-0.041	-0.042
					(0.953)	(0.792)	(0.541)	(0.614)	(0.848)	(0.588)	(0.467)
Mozambique	2001M12	0.0233	0.0089	0.1475	0.188	0.095	0.168	0.041	-0.146	0.090	-0.075
					(0.108)	(0.197)	(0.146)	(0.239)	(0.152)	(0.158)	(0.108)

Correlation Matrix

	South Africa	Swaziland	Namibia	Mozambique
South Africa	1			
Swaziland	0.165	1		
Namibia	0.042	-0.162	1	
Mozambique	-0.0343	-0.186	-0.262	1

Source: compiled by authors for South Africa from Datastream, Namibia from Standard & Poors (Frontier Market series) and Swaziland and Mozambique direct from national stock exchanges. All data reported in South African Rand end of period values. Values in parentheses for autocorrelations represent respective probabilities
.

Table 2. Descriptive Statistics

Country	South Africa	Swaziland	Namibia	Mozambique
Observations	207	207	105	70
Mean	0.0159	0.0176	0.0107	0.0233
Median	0.0179	0.0018	0.0189	-0.0085
Maximum	0.2022	0.5137	0.1491	0.4291
Minimum	-0.3234	-0.4001	-0.1651	-0.3979
Standard Deviation	0.0594	0.0982	0.0634	0.1475
Skewness	-0.7637	1.1809	-0.1211	0.4339
Kurtosis	7.7322	9.8521	3.1696	4.3031
Jarque Bera	213.27	453.06	0.3825	7.1491
Probability	0.0000	0.0000	0.8259	0.0280
Unit Root Test (ADF)	-14.24***	-13.59***	-10.04***	-6.48***

*** indicates hypothesis of unit root rejected at 99% confidence levels. Critical values at 90%, 95% and 99% significance levels are: -2.5896, -2.9042, -3.5285

Table 3. GARCH model parameters

	Period: 1990M07 – 2007M09				Period: 1999M01 – 2007M09				Period: 2001M12 – 2007M09			
	Coefficient	**Z-statistic**	**Coefficient**	**Z-statistic**	**Coefficient**	**Z-statistic**	**Coefficient**	**Z-statistic**	**Coefficient**	**Z-statistic**	**Coefficient**	**Z-statistic**
	South Africa (4,4)		Swaziland (9,6)		South Africa (2,6)		Namibia (1,1)		South Africa (3,1)		Mozambique (1,1)	
Mean Equation												
α	0.017	5.155 (0.000)	0.008	2.303 (0.021)	0.019	3.750 (0.0002)	0.011	2318.058 (0.000)	0.019	4.786 (0.000)	-0.009	-0.603 (0.546)
β	-0.065	-1.118 (0.264)	0.142	1.609 (0.107)	-0.034	-0.393 (0.6942)	0.067	0.613 (0.539)	-0.074	-0.746 (0.456)	0.041	24.436 (0.000)
GARCH representation for Residuals												
ω_0	0.0017	2.8649 (0.0042)	0.000152	2.0892 (0.0367)	0.00328	0.90942 (0.3631)	0.00016	1.2286 (0.2192)	0.00166	1.3478 (0.1777)	0.000193	0.7077 (0.4791)
δ1	0.059	0.419 (0.674)	1.112	4.084 (0)	-0.201	-0.209 (0.834)	1.095	14.240 (0.000)	-0.0094	-0.217 (0.828)	1.167	12.655 (0.000)
δ2	-0.214	-1.688 (0.091)	-0.131	-0.354 (0.722)	-0.399	-0.462 (0.644)	--	--	--	--	--	--
δ3	-0.038	-0.255 (0.798)	0.045	0.250 (0.801)	-0.394	-1.071 (0.283)	--	--	--	--	--	--
δ4	0.199	1.824 (0.068)	-0.409	-2.653 (0.008)	0.592	1.255 (0.209)	--	--	--	--	--	--
δ5	--	--	-0.137	-0.525 (0.599)	0.124	0.163 (0.870)	--	--	--	--	--	--
δ6	--	--	0.357	1.983 (0.047)	-0.166	-0.151 (0.879)	--	--	--	--	--	--
δ7	--	--	--	--	--	--	--	--	--	--	--	--
δ8	--	--	--	--	--	--	--	--	--	--	--	--
δ9	--	--	--	--	--	--	--	--	--	--	--	--
γ1	-0.055	-3.084 (0.002)	0.224	3.017 (0.003)	-0.002	-0.032 (0.973)	-0.125	-2.776 (0.005)	0.4538	4.258 (0.0000)	-0.132	-2.114 (0.034)
γ2	0.140	1.791 (0.073)	-0.227	-2.367 (0.017)	0.091	1.225 (0.220)	--	--	0.5404	11.723 (0.0000)	--	--
γ3	0.146	1.431 (0.152)	-0.0007	-0.008 (0.993)	--	--	--	--	-1.0191	-28.781 (0.0000)	--	--
γ4	0.328	2.466 (0.013)	0.019	0.243 (0.808)	--	--	--	--	--	--	--	--
γ5	--	--	0.102	1.168 (0.242)	--	--	--	--	--	--	--	--
γ6	--	--	0.122	1.331 (0.183)	--	--	--	--	--	--	--	--
γ7	--	--	0.017	0.162 (0.871)	--	--	--	--	--	--	--	--
γ8	--	--	-0.202	-1.837 (0.066)	--	--	--	--	--	--	--	--
γ9	--	--	0.106	1.938 (0.052)	--	--	--	--	--	--	--	--

GARCH lag order chosen via Akaike informational criterion (AIC). Values in parentheses are probabilities. Z-statistic significance levels are as follows: 60% confidence is 0.84; 70% confidence is 1.04; 80% confidence is 1.28; 90% confidence is 1.65; 95% confidence is 1.96; 99% confidence is 2.58.

Table 4. CAPM with GARCH error representation model parameters

	Period: 1990M07 – 2007M09				Period: 1999M01 – 2007M09				Period: 2001M12 – 2007M09			
	Coefficient	**Z-statistic**	**Coefficient**	**Z-statistic**	**Coefficient**	**Z-statistic**	**Coefficient**	**Z-statistic**	**Coefficient**	**Z-statistic**	**Coefficient**	**Z-statistic**
	South Africa (2,6)		Swaziland (7,9)		South Africa (1,4)		Namibia (1,1)		South Africa (1,1)		Mozambique (2,1)	
Mean Equation												
α_{Jenson}	0.0159	4.8638 (0.000)	0.0126	2.0183 (0.0436)	0.0162	4.955 (0.000)	0.0142	32.164 (0.000)	0.0132	2.810 (0.005)	-0.0086	-0.871 (0.384)
β	0.4307	12.575 (0.000)	0.0849	1.0687 (0.2852)	0.7114	10.786 (0.000)	-0.1721	-1.719 (0.085)	0.6506	7.503 (0.000)	0.5505	2.941 (0.003)
GARCH representation for Residuals												
ω_0	0.001126	1.7637 (0.0778)	0.002528	2.0675 (0.0387)	0.00194	2.0863 (0.0369)	0.000192	1.0524 (0.2926)	0.000672	0.5807 (0.5614)	0.000141	1.0636 (0.2875)
$\delta 1$	0.5157	0.675 (0.4997)	0.1175	0.2034 (0.8388)	-0.0151	-0.0641 (0.9489)	1.0586	14.314 (0.000)	0.4039	0.397 (0.6913)	1.202	8.872 (0.000)
$\delta 2$	-0.1949	-0.1869 (0.8517)	0.0434	0.0715 (0.9430)	-0.2056	-1.5539 (0.1202)	--	--	--	--	--	--
$\delta 3$	-0.1321	-0.1702 (0.8649)	0.1270	0.2383 (0.8116)	0.1737	0.8922 (0.372)	--	--	--	--	--	--
$\delta 4$	-0.0539	-0.1697 (0.8653)	-0.0261	-0.0599 (0.9522)	-0.5077	-1.4154 (0.1569)	--	--	--	--	--	--
$\delta 5$	0.0524	0.1546 (0.8771)	-0.3486	-1.0413 (0.2977)	--	--	--	--	--	--	--	--
$\delta 6$	0.0294	0.1258 (0.8999)	-0.1857	-0.5855 (0.5582)	--	--	--	--	--	--	--	--
$\delta 7$	--	--	0.0529	0.3272 (0.7435)	--	--	--	--	--	--	--	--
$\delta 8$	--	--	0.1079	0.9627 (0.3357)	--	--	--	--	--	--	--	--
$\delta 9$	--	--	0.2538	1.9891 (0.0467)	--	--	--	--	--	--	--	--
$\gamma 1$	0.1209	0.9368 (0.3488)	0.2091	4.3211 (0.0000)	0.3571	1.695 (0.09)	-0.0966	-2.822 (0.0048)	0.0916	0.3114 (0.756)	0.658	1.412 (0.158)
$\gamma 2$	0.2404	1.2243 (0.2208)	-0.0277	-0.2088 (0.8345)	--	--	--	--	--	--	-0.894	-1.762 (0.078)
$\gamma 3$	--	--	0.0541	0.3746 (0.7079)	--	--	--	--	--	--	--	--
$\gamma 4$	--	--	-0.0432	-0.2666 (0.7897)	--	--	--	--	--	--	--	--
$\gamma 5$	--	--	0.0331	0.2122 (0.8319)	--	--	--	--	--	--	--	--
$\gamma 6$	--	--	0.1356	0.9255 (0.3547)	--	--	--	--	--	--	--	--
$\gamma 7$	--	--	0.1057	1.1057 (0.2689)	--	--	--	--	--	--	--	--

GARCH lag order chosen via Akaike informational criterion (AIC). Values in parentheses are probabilities. Z-statistic significance levels are as follows: 60% confidence is 0.84; 70% confidence is 1.04; 80% confidence is 1.28; 90% confidence is 1.65; 95% confidence is 1.96; 99% confidence is 2.58.

Table 5. Annual average asset weights of each country in two-asset portfolio depending on strategy

Country	1992	1993	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007
Panel 1: Unconditional iid Strategy																
South Africa	----	----	----	----	----	----	----	----	----	22.03	44.37	54.06	62.85	70.31	68.70	62.11
Namibia	----	----	----	----	----	----	----	----	----	77.97	55.63	45.94	37.15	29.69	31.30	37.89
South Africa	----	9.41	30.33	42.55	67.65	89.37	87.29	77.55	72.28	59.59	83.49	85.35	84.42	81.34	61.45	52.00
Swaziland	----	90.59	69.67	57.45	32.35	10.63	12.71	22.45	27.72	40.41	16.51	14.65	15.58	18.66	38.55	48.00
South Africa	----	----	----	----	----	----	----	----	----	----	----	----	86.76	86.97	86.55	89.41
Mozambique	----	----	----	----	----	----	----	----	----	----	----	----	13.23	13.02	13.45	10.59
Panel 2: Conditional GARCH strategy																
South Africa	----	----	----	----	----	----	----	----	----	37.67	53.60	55.73	65.41	76.35	59.62	56.50
Namibia	----	----	----	----	----	----	----	----	----	62.33	46.40	44.27	34.59	23.65	40.38	43.50
South Africa	----	24.99	32.10	44.25	70.79	99.76	99.60	91.46	82.97	65.25	92.71	93.23	81.89	79.30	52.79	41.80
Swaziland	----	75.01	67.90	55.75	29.21	0.24	0.40	8.54	17.03	34.75	7.29	6.77	18.11	20.70	47.21	58.20
South Africa	----	----	----	----	----	----	----	----	----	----	----	----	80.95	88.64	91.13	93.71
Mozambique	----	----	----	----	----	----	----	----	----	----	----	----	19.05	11.36	8.87	6.29
Panel 3: Conditional CAPM-GARCH Strategy																
South Africa	----	----	----	----	----	----	----	----	----	76.48	80.60	76.52	84.13	98.54	88.95	74.91
Namibia	----	----	----	----	----	----	----	----	----	23.52	19.40	23.48	15.87	1.46	11.05	25.09
South Africa	----	24.99	32.10	44.25	70.79	99.76	99.60	91.46	82.97	65.25	92.71	93.23	81.89	79.30	52.79	41.80
Swaziland	----	75.01	67.90	55.75	29.21	0.24	0.40	8.54	17.03	34.75	7.29	6.77	18.11	20.70	47.21	58.20
South Africa	----	----	----	----	----	----	----	----	----	----	----	----	89.55	93.83	94.93	96.13
Mozambique	----	----	----	----	----	----	----	----	----	----	----	----	10.45	6.17	5.07	3.87

Values reported are the annual average asset weights which are obtained from each month's recursive quadratic portfolio optimisation process.

Table 6. Performance of unconditional and conditional strategies

Universe	Mean return (Annualized)	Standard Deviation (Annualized)	Minimum	Maximum
Strategy: Unconditional (iid model)				
South Africa - Swaziland	18.94	82.11	-0.22	47.79
South Africa – Namibia	15.24	66.67	-16.63	42.00
South Africa - Mozambique	29.08	73.27	-0.32	53.08
Strategy: Conditional (GARCH model)				
South Africa - Swaziland	19.30	87.14	3.11	58.04
South Africa – Namibia	21.21	68.49	12.36	27.91
South Africa - Mozambique	33.12	52.63	-20.21	131.93
Strategy: Conditional (CAPM model)				
South Africa - Swaziland	36.28	64.59	-67.18	212.58
South Africa – Namibia	43.76	56.47	-31.93	412.41
South Africa - Mozambique	44.62	50.33	-19.25	217.37

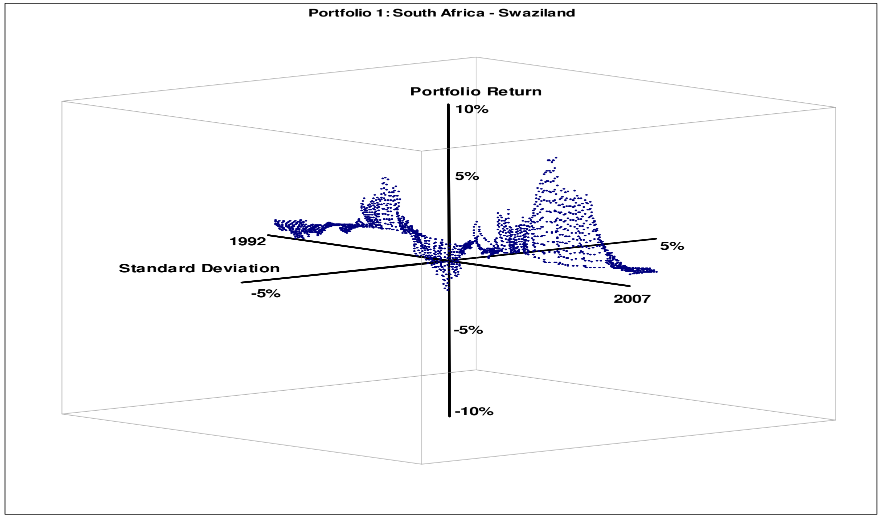
Table 7. Annual comparison of cost premiums derived from 30% minimum holding of micro-market alongside South Africa (Basis Points)

Year	Unconditional (iid model)			Conditional (GARCH model)			Conditional (CAPM model)		
	South Africa - Swaziland Geometric Mean	South Africa - Namibia Geometric Mean	South Africa - Mozambique Geometric Mean	South Africa - Swaziland Geometric Mean	South Africa - Namibia Geometric Mean	South Africa - Mozambique Geometric Mean	South Africa - Swaziland Geometric Mean	South Africa - Namibia Geometric Mean	South Africa - Mozambique Geometric Mean
1993	0.00	--	--	0.00	--	--	0.00		
1994	0.00	--	--	0.00	--	--	0.00		
1995	0.00	--	--	0.00	--	--	0.00		
1996	77.80	--	--	15.85	--	--	-0.04		
1997	462.33	--	--	37.18	--	--	-0.70		
1998	-65.11	--	--	-719.91	--	--	1.10		
1999	-167.88	--	--	-194.42	--	--	-2.89		
2000	-45.64	0.00	--	-112.35	--	--	-1.05	0.00	
2001	20.11	0.00	--	-8.04	0.00	--	0.91	-1.71	
2002	235.83	0.00	--	154.68	0.00	--	-1.05	0.81	
2003	760.70	0.00	-614.13	0.84	0.00	-196.09	0.19	-1.40	-37.42
2004	894.47	0.00	-613.37	5.96	0.00	-346.09	-0.91	-1.63	-26.88
2005	362.80	-13.80	-439.45	-67.06	-40.68	-619.29	-2.46	-9.41	-40.83
2006	0.00	-9.91	-101.51	-14.63	0.00	-526.15	-0.74	-3.65	-37.34
2007	0.00	0.00	567.95	0.00	0.00	-477.69	0.00	-0.07	-32.78

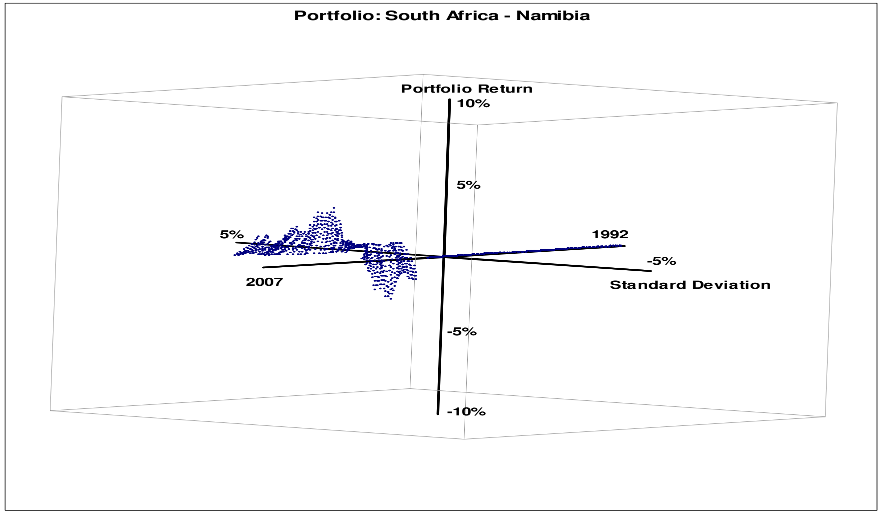
Values represent the difference or premium of holding a minimum investment weight of 30% over a 0% holding of the micro market within the two asset universe in the optimisation problem. Geometric mean is the annualised geometric mean for each respective year in Basis points.

Figure 1. Unconditional asset allocation (no short sales)

1(a) South Africa – Swaziland portfolio (12/1992 – 07/2007)



1(b) South Africa – Namibia portfolio (12/2000 - 07/2007)



1(c) South Africa – Mozambique portfolio (12/2003 – 07/2007)

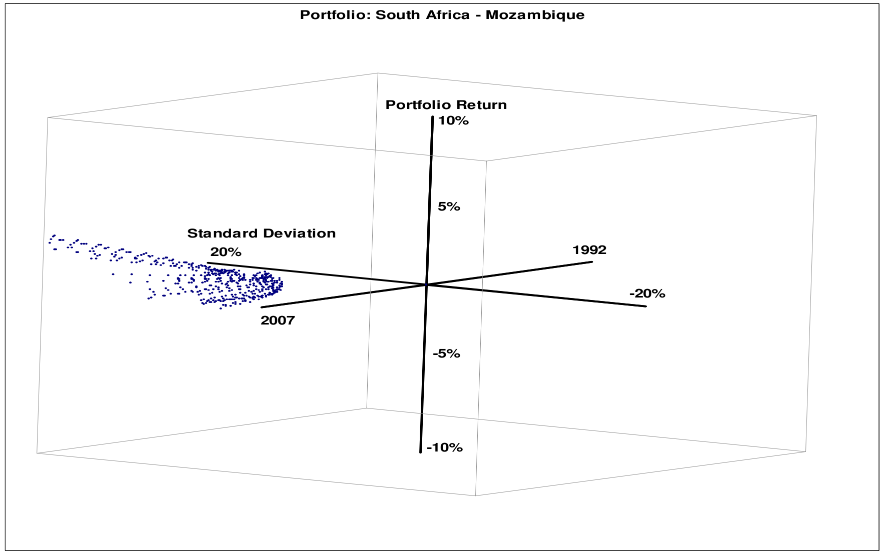
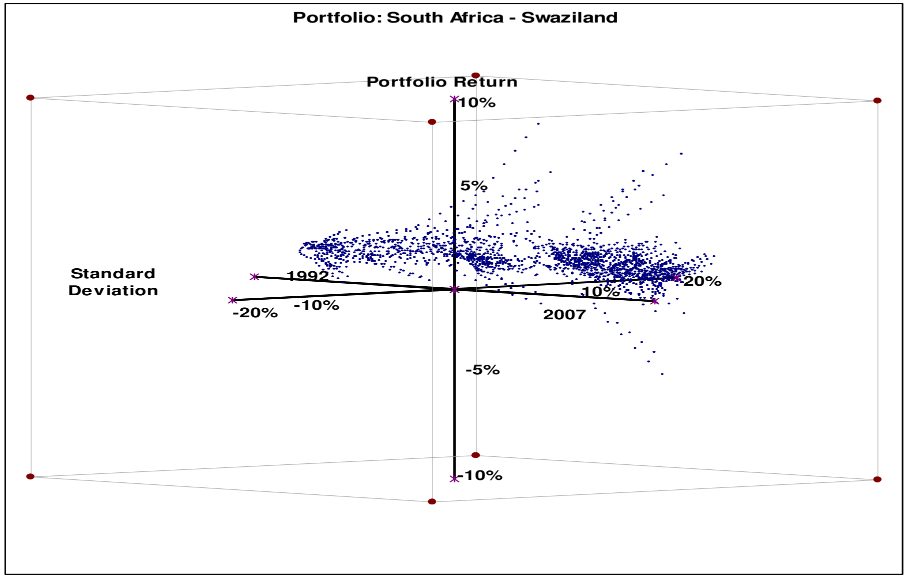
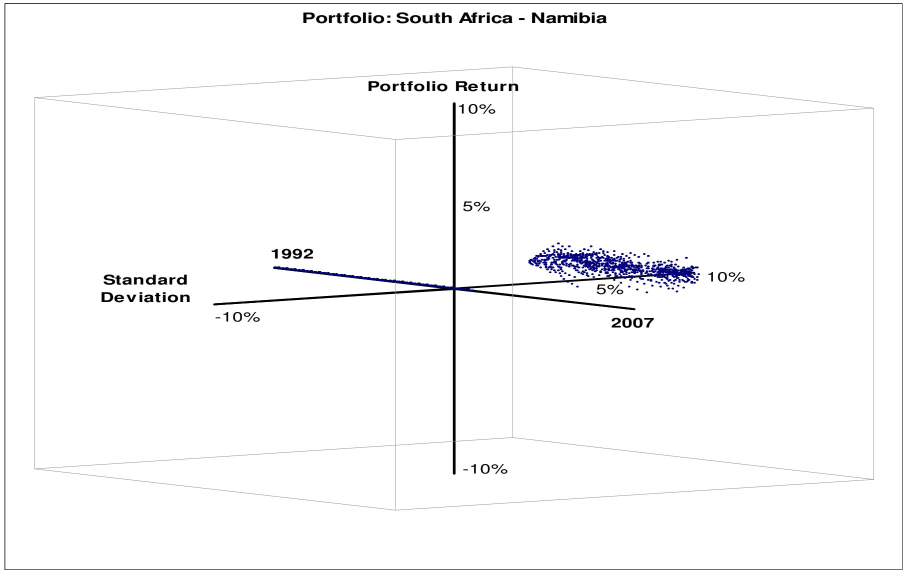


Figure 2. Conditional asset allocation using GARCH model (no short sales)

2(a) South Africa – Swaziland portfolio (12/1992 – 07/2007)



2(b) South Africa – Namibia portfolio (12/2000 - 07/2007)



2(c) South Africa – Mozambique portfolio (12/2003 – 07/2007)

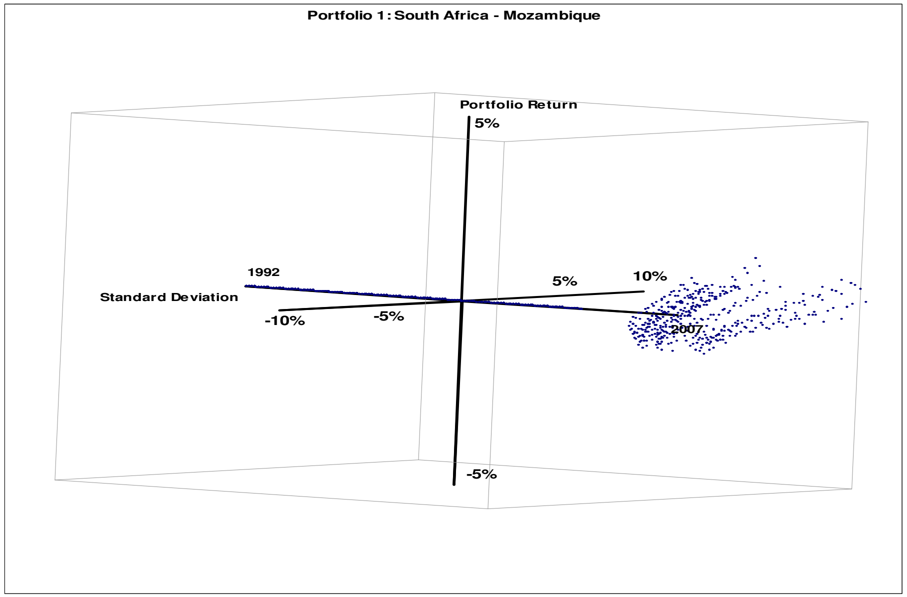
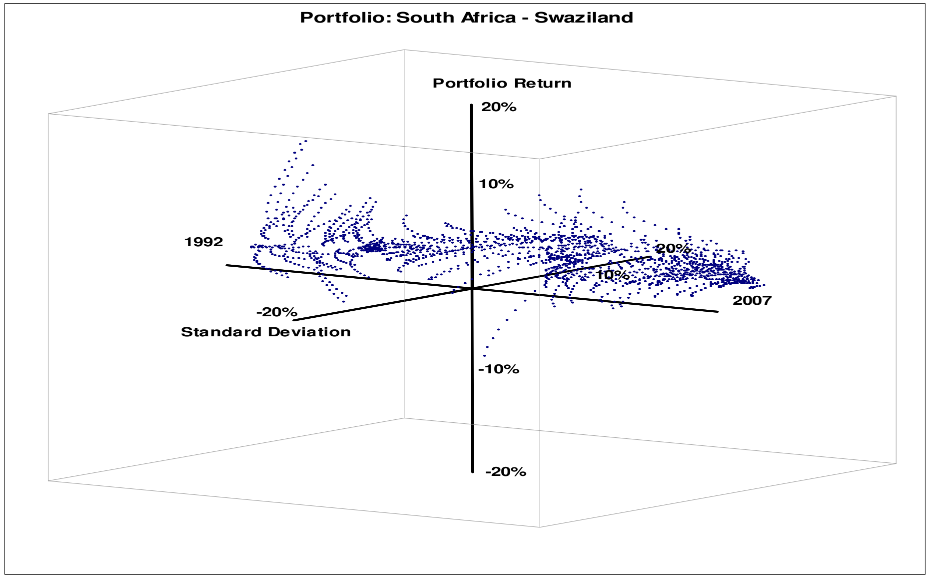
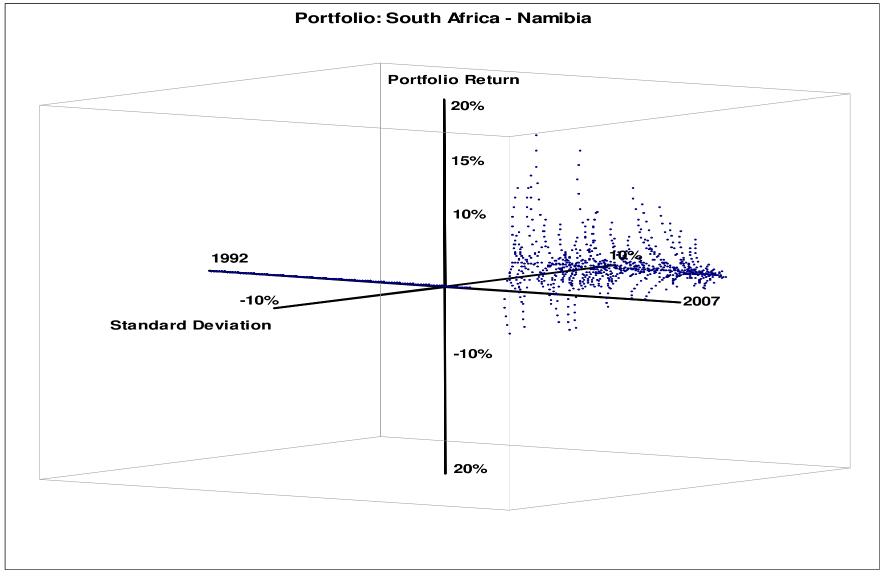


Figure 3. Conditional asset allocation using CAPM model (no short sales)

3(a) South Africa – Swaziland portfolio (12/1992 – 07/2007)



3(b) South Africa – Namibia portfolio (12/2000 - 07/2007)



3(c) South Africa – Mozambique portfolio (12/2003 – 07/2007)