

# Forecasting a Small Open Economy: NKDSGE versus DFM

Rangan Gupta

Associate Professor

University of Pretoria

and

Alain Kabundi

Senior Lecturer

University of Johannesburg

## Objective:

We analyze the ability of a *SOE NKDSGE* model relative to a *DFM*, in forecasting the *Per Capita Growth Rate*, *CPI Inflation*, *Money Market Rate*, and the *Growth Rate of the Nominal Effective Exchange Rate*,

By estimating the models using South African Quarterly data over the period of **1983:Q1 to 2002:Q4**, and;

Forecasting *one- to four-quarters-ahead* over the out-of-sample horizon of **2003:Q1 to 2006:Q4 (First Attempt)**.

## Motivation:

- Gupta and Kabundi (2008): DFM outperforms a closed economy NKDSGE model (Gupta *et al.* (2007)) in terms of forecasting Per Capita Growth Rate, GDP Deflator Inflation and the 91 Days Treasury Bill Rate;
- Ortiz and Sturzenegger (2007): Estimating SARB's Policy Reaction Function (1983:Q1-2002:Q4) based on the Lubik and Schorfheide (2007) SOE NKDSGE model  $\Rightarrow$  Stable Rule with a consistent anti-inflation bias;
- How about **Forecasting** using the model based on the choice of priors (Bayesian Estimation) by Ortiz and Sturzenegger (2007)?

## The SOE NKDSGE Model:

- Three-Equation RATEST Model: Open-Economy IS Curve, Open-Economy Phillips Curve, and Monetary Policy Rule;
- Open Economy IS Curve derived from the Euler Equation of Consumer Maximization (and Aggregate Demand Matters  $\Rightarrow$  Monopolistic Competition);
- The Phillips Curve originates from the assumption of Price Rigidities (Calvo's (1983) Price Staggering Mechanism);
- Monetary Policy described by an Interest Rate Rule.

## The SOE NKDSGE Model (Continued):

### Open-Economy IS Curve:

$$y_t = \left\{ \begin{array}{l} E_t y_{t+1} - [\tau + \alpha(2 - \alpha)(1 - \tau)](R_t - E_t \pi_{t+1}) - \rho_z z_t \\ -\alpha[\tau + \alpha(2 - \alpha)(1 - \tau)]E_t \Delta q_{t+1} + \alpha(2 - \alpha)\frac{1 - \tau}{\tau} E_t \Delta y_{t+1}^* \end{array} \right\} (1)$$

where,  $y_t$ : Real GDP;  $R_t$ : Nominal Money Market Rate;  $\pi_t$ : CPI Inflation;  $z_t$ : Growth Rate of a Non-Stationary World Technology Process  $Z_t$ ;  $q_t$ : TOT (Relative Price of Exports to Imports);  $y_t^*$ : Exogenous World Output;  $\tau$ : Elasticity of Intertemporal Substitution;  $\alpha$ : Import Share;  $\rho_z$ : AR Coefficient of  $z_t$ .

To ensure Stationarity: All Real variables expressed as percentage deviations from  $Z_t$ .

## The SOE NKDSGE Model (Continued):

### Open-Economy Phillips Curve:

$$\pi_t = \beta E_t \pi_{t+1} + \alpha \beta \Delta q_{t+1} - \alpha \Delta q_t + \frac{\kappa}{\tau + \alpha(2 - \alpha)(1 - \tau)} (y_t - \bar{y}_t) \quad (2)$$

where,  $\bar{y}_t = -\alpha(2 - \alpha) \frac{1 - \tau}{\tau} y_t^*$ : Potential Output (in absence of Nominal Rigidities);  $\beta$ : Discount Factor;  $\kappa$ : Structural Parameter (defining the Slope of the Phillips Curve)

### Interest Rate Rule:

$$R_t = \rho_R R_{t-1} + (1 - \rho_R) [\psi_1 \pi_t + \psi_2 y_t + \psi_3 \Delta s_t] + \epsilon_t^R; \quad \epsilon_t^R \rightarrow N(0, \sigma_R^2) \quad (3)$$

where,  $\rho_R$ : Persistence in the Interest Rate;  $s_t$ : Nominal Effective Exchange Rate;  $\psi_i$ ,  $i = 1, 2, 3$ : Monetary Authority's Reaction Parameter to inflation, output and exchange rate fluctuations.

## The SOE NKDSGE Model (Continued):

The Exchange Rate is introduced via the CPI Inflation according to:

$$\pi_t = \Delta s_t + (1 - \alpha)\Delta q_t + \pi_t^* \quad (4)$$

where,  $\pi_t^*$ : World Inflation Shock.

Growth Rate in TOT follows an Exogenous Process:

$$\Delta q_t = \rho_q \Delta q_{t-1} + \epsilon_t^q; \quad \epsilon_t^q \rightarrow N(0, \sigma_q^2) \quad (5)$$

In addition:

$$y_t^* = \rho_{y^*} y_{t-1}^* + \epsilon_t^{y^*}; \quad \epsilon_t^{y^*} \rightarrow N(0, \sigma_{y^*}^2) \quad (6)$$

$$\pi_t^* = \rho_{\pi^*} \pi_{t-1}^* + \epsilon_t^{\pi^*}; \quad \epsilon_t^{\pi^*} \rightarrow N(0, \sigma_{\pi^*}^2) \quad (7)$$

## Estimation:

The SOE NKDSGE model presented above is estimated using Bayesian Methods. The object of interest is the vector of parameters:

$$\theta = (\psi_1, \psi_2, \psi_3, \alpha, \beta, \kappa, \tau, \rho_q, \rho_z, \rho_{y^*}, \rho_{\pi^*}, \rho_R, \sigma_R, \sigma_q, \sigma_z, \sigma_{y^*}, \sigma_{\pi^*})$$

Given a prior  $p(\theta)$ , the posterior density of  $\theta$  is:  $p(\theta|Y^T) = \frac{L(\theta|Y^T)p(\theta)}{\int L(\theta|Y^T)p(\theta)d\theta}$ , where  $L(\theta|Y^T)$ : Likelihood conditional on observed data  $Y^T = Y_1, \dots, Y_T$ .

In our case,  $Y_t = [\Delta y_t + z_t, 4\pi_t, 4R_t, \Delta s_t, \Delta q_t]'$ .

## Estimation (Continued):

- (i)  $L(\theta|Y^T)$  computed by combining the state-space representation obtained from the RATEX solution of the model and the Kalman filter, based on normally distributed errors;
- (ii) The Posterior draws are obtained from MCMC method;
- (iii) After obtaining the Mode of the Posterior, A Random Walk Metropolis Algorithm is used to generate Posterior Draws;
- (iv) Point estimates and measures of uncertainty for  $\theta$  are obtained from the generated values;
- (v) Once estimated over 1983:Q1-2002:Q4, we recursively estimate over 2003:Q1-2006:Q4 to generate one- to four-quarters ahead forecasts.

## Priors:

Based on previous estimations and available information (Ortiz and Sturzenegger (2007)):

$\psi_1 \rightarrow \text{Gamma}(1.50, 0.50)$ ,  $\psi_2 \rightarrow \text{Gamma}(0.25, 0.13)$ ,  
 $\psi_3 \rightarrow \text{Gamma}(0.90, 0.50)$ ,  $\alpha \rightarrow \text{Beta}(0.30, 0.05)$ ,  
 $r$  (real interest rate)  $\rightarrow \text{Gamma}(2.50, 1.50)$ ,  $\kappa \rightarrow \text{Gamma}(0.80, 0.30)$ ,  
 $\tau \rightarrow \text{Beta}(0.50, 0.20)$ ,  $\rho_q \rightarrow \text{Beta}(0.60, 0.20)$ ,  
 $\rho_z \rightarrow \text{Beta}(0.30, 0.07)$ ,  $\rho_{y^*} \rightarrow \text{Beta}(0.90, 0.05)$ ,  
 $\rho_{\pi^*} \rightarrow \text{Beta}(0.40, 0.10)$ ,  $\rho_R \rightarrow \text{Uniform}(0.50, 0.20)$ ,  
 $\sigma_R \rightarrow \text{Invgamma}(0.50, 4.00)$ ,  $\sigma_q \rightarrow \text{Invgamma}(4.50, 4.00)$ ,  
 $\sigma_z \rightarrow \text{Invgamma}(1.00, 4.00)$ ,  $\sigma_{y^*} \rightarrow \text{Invgamma}(1.50, 4.00)$ ,  
 $\sigma_{\pi^*} \rightarrow \text{Invgamma}(2.50, 4.00)$ .

# DFM

- In a DFM, each time series in the panel is represented as the sum of two latent components:
  - a common component which captures most of the multivariate correlation and
  - an idiosyncratic component which is poorly cross-sectionally correlated
- The rationale behind factor analysis is that common components are driven by a few common shocks
- Such low dimensionality implies that common components can be consistently estimated and forecasted on the basis of few factors only

## DFM (Cont.)

- Dynamic form:  $x_{it} = b_i(L)f_t + \xi_{it} = \chi_{it} + \xi_{it}$
- Static form:  $x_{it} = \lambda_i F_t + \xi_{it} = \chi_{it} + \xi_{it}$
- Forni et al. (2005) estimate dynamic factors through the use of dynamic principal component analysis
- It involves the estimating the eigenvalues and eigenvectors decomposition of spectral density matrix of  $X_t$

## DFM (Cont.)

- Spectral density:

$$\Sigma_x(\lambda) = \Sigma_\chi(\lambda) + \Sigma_\xi(\lambda)$$

where  $\lambda \in [-\pi, \pi]$

- Assumptions:

1. The process  $f_t$  and  $\xi_{it}$  are orthogonal at all leads and lags
2.  $\mu_{n,q}^\chi(\lambda) \rightarrow \infty$  in  $[-\pi, \pi]$
3.  $\mu_{n1}^\xi(\lambda) \leq M$  for any  $n$  and  $\lambda$  in  $[-\pi, \pi]$

## DFM (Cont.)

- MA Assumptions:

1.  $b_i(L) = b_{i,0} + b_{i,1}L + b_{i,2}L^2 + \dots + b_{i,s}L^s$  .

2.  $f_{t-j}$  and  $\xi_{i,t-j}$  are orthogonal

3.  $\mu_{nr}^\chi \rightarrow \infty$  where  $r = q(s+1)$

4.  $\mu_{n1}^\xi(\lambda) \leq M$  for any  $n$  and  $\lambda$  in  $[-\pi, \pi]$

## Data used for the DFM:

- The data set contains 267 quarterly series:  
real, nominal, and financial sectors;  
intangible variables, such as confidence indices, and survey variables;  
global variables such as commodity industrial inputs price index and crude oil prices;  
series of major trading partners such as Germany, the UK, and the US;
- The in-sample period contains data from 1983Q1 to 2002Q4;
- The out-of-sample set is 2003Q1-2006Q4.

## Conclusions and Areas of Further Research:

- THE SOE NKDSGE model outperforms the DFM as far as forecasting inflation, interest rate, and growth rate of exchange rate is concerned;
- DFM outperforms the SOE NKDSGE in terms of the forecasts of growth rate per capita;
- More work on the SOE NKDSGE required (Habit Persistence, Wage Rigidity, Imperfect Pass-Through, Import Price Rigidity, etc.), [Heterogeneity (Better Modelling of the Informal Sector and the Labor Market) (Market Segmentation Models?)].