Estimation of hedge ratios in the South African futures exchange

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Abstract

The primary objective for a risk-averse investor in the portfolio theory is to attain minimum-variance from a futures hedge strategy undertaken over a period with the investor predominantly 'short' in futures. The Ordinary Least Square (OLS) regression model, Error Correction Model (ECM), Vector Error Correction Model (VECM) and the Generalised Autoregressive Conditional Heterescedasticity (GARCH 1,1) with error correction model are specified to estimate the risk-minimising hedge ratio. Focus is on the FTSE/JSE Top 40 index and its corresponding index futures in the South African futures exchange. It is discovered that the ECM and VECM models provide an equivalent hedge ratio although just like the OLS model, they offer an under-estimate of the spot position to be hedged. The ECM-GARCH (1, 1) dominates the other models with the largest hedge ratio hence it is potentially more efficient in reducing the risk of the spot price.

Chapter 1

Introduction

Stock index futures are one type of financial derivatives used successfully in risk management because of their relatively high degree of liquidity, low transaction costs and simplified credit risk. Specifically, futures offer investors an incentive to hedge asset portfolios by undertaking opposite transactions or positions in the cash (spot) and futures markets in order to reduce price risk exposure in the spot market. Assuming an investor purchases (long) units of stock index in the spot market that is susceptible to unfavourable price fluctuations, the investor's unforeseen loss can be compensated through a gain from favourable price movements in the futures contracts sold (short). Therefore, an estimate of the number of futures contracts[1] sold relative to the number of units of the stock index is required. The estimate is the hedge ratio and its influence on the risk reduction achieved can depend on the estimation technique adopted.

The preceding account is cautious because of the different perspectives to the nature and purpose of hedging ranging from; speculating on changes in the basis for profit-maximisation (Working, 1953), anticipated risk-return trade-off in an investors portfolio (Johnson, 1960) as well risk-minimisation in a portfolio of assets held by an investor (Ederington, 1979). The latter perspective is pivotal in this study while discussion on different sections of this work highlights key issues from the first two. It is also considered that the perspectives are originally based on hedging in commodity futures markets yet they are also applicable to hedging in index futures markets, which is the focus of this research.

Risk-minimisation in the portfolio theory applied through the minimum-variance hedge ratio (MVHR) has become universal in use, as it has well known statistical properties. This assumes minimising the price risk that is defined as the variance or standard deviation of a subjective distribution for price change over a given time

[1] A future contract is an agreement that obligates the holder to buy or sell an asset at a predetermined delivery price during a specified future time period. The exchange takes place in organised futures markets where the risk of price changes in an asset is transferred through a futures contract from investors to speculators more willing to bear the risk (Sutcliffe, 1997; Hull, 1997)

period, that a trader holds at the initial time, where actual price change over the period is treated as a random variable (Johnson, 1960). This hedge ratio is assumed to be optimal in a traditional Ordinary Least Square (OLS) regression where it is obtained from the constant unconditional covariance between spot and futures prices divided by the constant unconditional variance of the futures prices (Ederington, 1979). While one of the arguments against the optimal assumption is that it does not reflect utility maximisation that depends on expected returns (Figlewski, 1984; Ceccheti, Cumby and Figlewski, 1988; Sephton, 1993; Brooks, Henry and Persand, 2002) some textbook definitions endorse it as the optimal hedge ratio in an unbiased market (Hull, 1997). One of its strength is that it recognises the imperfect correlation between spot and futures prices and supersedes other types of risk-minimising hedge ratios namely; the naïve or unit-for-unit and beta.

The naïve hedge ratio in historical futures hedging is negative one so that the futures position is equal in magnitude but opposite in sign to the spot position. This perfect hedge strategy assumes perfect correlation between stock index prices and index futures prices and a zero basis risk[2], and such that the risk of the price is eliminated but the cost of the hedge in the form of transaction costs (Sutcliffe, 1997; Hull, 1997). However, the volatility of the futures contract tends to be higher than the spot asset causing a non-zero basis risk and imperfect correlation between the spot and futures prices. It is for this reason that the naïve hedge ratio is ineffective.

Another hedge ratio which specifically captures systematic risk in a stock portfolio is termed a beta hedge. Beta (β) is based on the capital asset pricing model where it is the slope of the best-fit line obtained when excess return on the portfolio over the risk-free rate is regressed on the excess return of the market over the risk-free rate. It is assumed to deliver a hedge ratio equivalent to the naïve hedge ratio when the stock portfolio to be hedged matches the spot asset underlying the futures contract. In extension, this strategy also allows the use of the parameter beta (β) to estimate the appropriate hedge ratio when the stock portfolio to be hedged does not match the spot asset underlying the futures contract, a process termed cross-hedging (Sutcliffe, 1997;

[2] The basis risk is equivalent to the sum of the variance of the futures price and the variance of the spot price less twice the covariance between futures and spot prices, denoted as $\sigma^2_{F-S} = \sigma^2_F + \sigma^2_S - 2\sigma^2_{F,S}$ and it is only zero at the maturity of the futures contract.

Hull, 1997). Conversely, this beta hedge ratio ignores the daily cash settlement of the futures contract that cause fluctuations in the basis such that the ratio may not minimise risk but generate an over-hedged portfolio. Hence, empirical research demonstrates the superiority of the minimum-variance hedge ratio.

However, the application of the OLS to derive the hedge ratio has raised considerable criticism based on evidence that that it ignores the unique behaviour of asset prices and returns. The non-normal and fat-tailed distributions of returns alongside the variance that changes over time due to the impact of news in the market are the reasons for the use of the generalised autoregressive conditional heteroscedasticity (GARCH) model in estimating hedge ratios. This type of model incorporates the time-dependent conditional covariance and conditional variances thus generate an un-biased hedge ratio that reduces the risk effectively (Ceccheti, Cumby and Figlewski, 1988; Myers and Thompson, 1989; Baillie and Myers, 1991). Moreover, empirical evidence indicates that time-varying hedge ratios estimated with multivariate GARCH models facilitate the continuous adjustment of the hedge ratio and often yields better performance in terms of reducing the risk of a portfolio rather than a constant hedge ratio.

In addition, the spot and futures prices can be characterised by a long-run equilibrium relationship in certain cases yet the traditional OLS does not account for it. When ignored, this leads to the underestimation of the OLS hedge ratio translating to an inefficient risk-minimisation strategy. In that case, an ECM is utilised in order to capture the short-run dynamics and long-run information thus providing an optimal cointegrating hedge ratio than the OLS (Ghosh, 1993; Lien and Luo, 1993; Chou, Denis and Lee, 1996). A different view is that the ECM is too restrictive with regressors and fails to test hypothesis on the actual number of cointegration relationships. This limitation can be addressed by the VECM in which case a large hedge ratio from the VECM indicates that the ECM under-estimates the spot position to be hedged (Lien, 1996; Lien, 2004).

At this stage, an example[3] helps to illustrate the practical application of the risk-minimising hedge ratio. Assume that an institution invests in the Johannesburg Stock Exchange (JSE) that is a spot market, and specifically holds an equity portfolio (with weights equal to the FTSE/JSE Top 40) between today as 2nd January 2006 and 5th March 2006. The institution develops an objective to minimise the risk of its spot portfolio FTSE/JSE Top 40 which has a current market value of R58 million while the value or closing price on 2nd January 2006 of the March FTSE/JSE Top 40 index futures contract is of 2,400. If the amount of equity exposure inherent in one equity index futures contract is R23, then the current value of the portfolio is equal to the current price of 1,050 units of the spot index calculated as (58,000,000/(2,400 x 23)). With an expected unconditional covariance between the spot and futures price of R6,600 when the hedge is lifted (5th March) and an expected unconditional variance of the futures price of R8,500 on the same date, the risk-minimising hedge ratio is (R6,600/R8,500) = 0.78. Therefore, the number of FTSE/JSE Top 40 futures contracts that the institution has to sell in the South African Futures Exchange (SAFEX) in order to hedge the equity portfolio (1,050 x 0.78) = 819 contracts.

The objective of the study is to specify four econometric models and estimate hedge ratios to ascertain which model potentially offer the minimum risk of spot prices. The models in the following order; OLS, ECM, VECM and ECM-GARCH (1,1) are justified because there is evidence that they provide varying performance in risk minimisation for different index futures markets. Application is based the FTSE/JSE Top 40 stock index and its index futures contract in the South African Futures Exchange Market (SAFEX). The study has an advantage because the researcher did not find evidence of a similar comprehensive work undertaken for the same market. Moreover, the ECM-GARCH (1,1) is an improvement on univariate GARCH models specified by other authors in the past. This work has five chapters including this section. Chapter 2 provides a background on the determination of the minimum-variance hedge ratio in the portfolio theory. The chapter is extended to describe the methodology designed to meet the objectives of the study. Chapter 3 is a discussion of the relevant literature reviewed. The empirical results and analysis are presented in Chapter 4 with a conclusion in the last chapter.

[3] Example modified from Sutcliffe (1997).

Chapter 2

Theory and Methodology

2.1 Theory

The portfolio theory demonstrates that an investor holds a fraction (h) of the spot position in the futures market as the hedged assets and leaves a fraction (1 - h) as non-hedged assets while spot and futures markets are not substitutes (Ederington 1979). This view marks the difference from the naïve risk-minimisation theory which stipulates that investors' assets should always be fully hedged (h = 1); the profit-maximisation theory (Working, 1953) where an investor can either be fully hedged or non-hedged; and the traditional portfolio approach (Johnson, 1960). Ederington's portfolio theory assumes that there is no asset without risk because even treasury bills may bear a price risk when there is need to liquidate before maturity. An additional assumption is that spot market holdings are exogenous such that any interest payment is viewed as predetermined, hence irrelevant to hedging decisions.

The primary objective is to reduce fluctuation in the value of the portfolio that is caused by market risk. In the first practical application of the portfolio theory, a risk-averse investor's objective is to attain minimum-variance from a futures hedge strategy (Ederington, 1979) undertaken over a period (t) with investors predominantly 'short' in futures. That is, selling x_f units in one futures contract in order to finance the purchase of given x_s units of asset or stock in the spot market. Then consider S_t and F_t as the random or stochastic stock index (spot) price and random contracted stock index futures price such that the expected returns[4] from each asset at time t are stated respectively as;

$$\Delta S_t = S_t - S_{t-1} \tag{i}$$

$$\Delta F_t = F_t - F_{t-1} \tag{ii}$$

[4] The statistical derivation of 'expected' returns is not presented in order to allow for the model specifications at a later stage where the expected returns are simply the first difference of logarithmic spot and futures prices.

The expected return on a long spot position ($\Delta P_{s,t}$) is a product of the units purchased and the expected price change at time t equation (iii) while equation (iv) represents the variance ($\sigma^2_{s,t}$) of the same non-hedged position;

$$\Delta P_{s,t} = x_s[\, S_t - S_{t-1}] \qquad \text{(iii)}$$

$$\sigma^2_{s,t} = x^2_s \sigma^2_s \qquad \text{(iv)}$$

Similarly, expected return on the short futures position ($\Delta P_{f,t}$) and its corresponding variance ($\sigma^2_{f,t}$) are shown in equations (v) and (vi). While the rate of return on an index futures contract is generally equivalent to the total return on the underlying index portfolio, minus the dividend yield on the index plus the change in basis over the period as a fraction of the initial index, this breakdown is not applied in this study.

$$\Delta P_{f,t} = x_f[\, F_t - F_{t-1}] \qquad \text{(v)}$$

$$\sigma^2_{f,t} = x^2_f \sigma^2_f \qquad \text{(vi)}$$

Therefore, the expected return on a fully or partially hedged portfolio ($\Delta P_{h,t}$) is a sum of the returns on non-hedged spot position and returns on futures position as shown in equation (vii) without incorporating any initial outlay of capital[5]. The conditional variances of possible price changes over time t of the spot position ($\sigma^2_{s,t}$) and futures position ($\sigma^2_{f,t}$) together with the conditional covariance of the returns on spot and futures in the same index ($cov_{sf,t}$) form the variance of the portfolio ($\sigma^2_{h,t}$) in equation (viii);

$$\Delta P_{h,t} = x_s[\, S_t - S_{t-1}] + x_f[F_t - F_{t-1}] \qquad \text{(vii)}$$

$$\sigma^2_{h,t} = x^2_s \sigma^2_s + x^2_f \sigma^2_f + 2\, x_s\, x_f\, cov_{sf,t} \qquad \text{(viii)}$$

A hedge ratio denoted by $h = -x_f/x_s$ is then assumed, representing the current value of the stock index sold in one futures contract as a fraction of the current value of the spot position being hedged. The negative sign demonstrates the hedging of a long spot position requires a short position in the corresponding futures contract while effectively h is positive.

[5] Figlewski (1984:659) argues that "the initial margin deposit to open a futures position does not represent an investmet of capital since it can be posted in the form of interest bearing Treasury bills." This statement marked one of the major differences in hedging portfolios with stock index futures rather than commodity futures.

Thus, the new expected return on a hedged portfolio ($\Delta P_{h,t}$) comprising one unit of the stock (spot) index and h units of the futures contract is written as equation (ix) and the variance or risk of the portfolio is equation (x);

$$\Delta P_{h,t} = x_s[(S_t - S_{t-1}) - h(F_t - F_{t-1})] \quad \text{(ix)}$$

$$\sigma^2_{h,t} = x^2_s\sigma^2_s + h^2\sigma^2_f - 2\,h\,\text{cov}_{sf,t} \quad \text{(x)}$$

Ultimately, the constant (time-invariant) risk-minimising hedge ratio (h^*) in equation (xi) is estimated by keeping x_s constant and setting the derivative of the variance of the return of the portfolio ($\sigma^2_{h,t}$) to zero with respect to h in equation (x) such that;

$$h^* = -\text{cov}_{sf}/\sigma^2_f \quad \text{(xi)}$$

In essence, h^* is defined as the unconditional covariance between cash and futures prices relative to the unconditional variance of the futures prices. It is the optimal hedge ratio obtained as the slope coefficient of the ordinary least square (OLS) regression, when ΔS_t is regressed on ΔS_t (Ederington, 1979). Since h^* is constant, investors can re-estimate it whenever market conditions change under restrictive assumptions. First, the expected return to holding a futures contract must be zero, that is follow a martingale process [$E(F_t - F_{t-1})=0$]. Second, the processes generating the covariance matrix of cash and future prices must be constant over time.

The optimal presumption for h^* is challenged because of the high degree of risk-aversion assumed and the lack of consideration for the portfolio's risk-return trade-off in a utility maximisation function. For that reason, the investor's utility can be estimated as a linear function of expected return on the hedged portfolio ($\Delta P_{h,t}$) and its variance ($\sigma^2_{h,t}$) as well as a risk-aversion parameter $\forall$ (Figlewski, 1984; Cecchetti, Cumby and Figlewski, 1988; Sephton, 1993; Brooks, Henry and Persand, 2002) in equation (xii);

$$U(\Delta P_{h,t}, \sigma^2_{h,t}) = \Delta P_{h,t} - \forall\sigma^2_{h,t} \quad \text{(xii)}$$

Equation (xii) is expanded to the show the investors' two-moment utility function in equation (xiii), where h in this case reflects the overall risk and return characteristics of the hedged position in the portfolio;

$$max\ U(\Delta P_{h,t}, \sigma^2_{h,t}) = x_s(\Delta S_t) - h(\Delta F_t) - \forall(x^2_s\sigma^2_s + h^2\sigma^2_f - 2\ h\ cov_{sf,t}) \qquad (xiii)$$

Nevertheless, it is still assumed that the risk-aversion parameter $\forall$ approaches infinity ($\propto$) and the martingale process holds, then the investors position does not depend on the risk parameter such that differentiating (xiii) with respect to h results in an optimal hedge ratio similar to h^*.

Substituting (h^*) into the variance of the portfolio in equation (x) results in the variance of returns for the minimum risk hedge in equation (xiv),

$$\sigma^2_{min} = \sigma^2_s(1 - \rho^2_{sf}) \qquad (xiv)$$

The correlation coefficient between the returns on the spot position and the futures contract is represented by ρ^2_{sf}. Equation (xiv) demonstrates that the return on a hedged portfolio will normally be subject to the risk caused by unanticipated changes in the relative price between the position being hedged and the futures contract. This expression also shows that price risk can only be eliminated with hedging in futures when there is perfect correlation between the returns on the spot position and the futures contract.

In another application the constant hedge ratio is written as h_0 in equation (xv) to reflect the correlation coefficient. That is the risk-minimising hedge ratio depends on the product of the correlation coefficient and the ratio of the standard deviation of the spot position to the standard deviation of the futures position.

$$h_0 = \rho(\sigma_s / \sigma_f) \qquad (xv)$$

The optimal minimum-variance hedge ratios h^* and h_0 can also be defined as beta (β) of the portfolio with respect to the futures contract. Such is the case only when dividends are not random and the hedge is to be held until the futures contract expire, yielding a stable change in the basis. Nonetheless beta is unlikely to be an optimal hedge ratio because the change in the basis is generally volatile. In summary, if the expected return to holding futures is zero, that is a none profit-maximising objective, then the minimum-variance hedging rule is also generally the expected utility-maximising hedging rule. Hence, the minimum variance hedge ratio is widely applicable to risk investors and underpins model specification in this study.

2.2 Methodology

2.2.1 Diagnosis of stylised facts

The evolution of various econometric models employed by numerous authors in the same subject under study is influenced by special characteristics of financial data termed ‘stylised facts’ (Pagan, 1996; Brooks, 2002). Hence, it is imperative to understand and identify them as they influence the effectiveness of the estimated hedge ratios in minimising the risk or variance of a portfolio. Raw stock index and stock index futures prices are transformed[6] with the natural logarithm operator in order to keep the series on the same relative scale while all hedge ratios are estimated from the logarithmic returns.

First, the unconditional distribution of the spot and futures prices and the respective returns is examined following evidence that it is crucial in constructing a model for optimal hedging (Baillie and Myers 1991). The Jarque-Bera test (1981) in Brooks, 2002; Gujarati, 2003 is employed. It is an asymptotic or large-sample test that follows a chi-square distribution with 2 degrees of freedom. The null hypothesis is that each of the series in normally distributed with zero mean and constant variance such that the JB test statistic would not be significant. The estimate of the kurtosis aids in determining the size or ‘fatness’ of the tails while skewness measures the degree of asymmetry of the distribution over its mean value. The latter two terms are used to substantiate the JB statistic and also describe third and fourth moments of the data. Measures of central tendency are also estimated alongside the standard deviation which is squared to derive the variance or estimate of volatility of prices.

[6] The econometric package used is Eviews version 5.0.

The second test involves the investigation of a unit root or nonstationarity in the logarithmic prices and their differences. The test is crucial because the efficient market hypothesis suggest that stock index and stock index futures prices should follow a random walk process or have a unit root while their differences or returns are generally stationary. Brooks (2002: 367) defines a stationary series as one with;

> "with a constant mean, constant variance and constant autocovariances for each given lag."

Testing for the unit root serves varying aspects of this study. The possibility of a long-run relationship or cointegration between logarithmic spot and futures prices can be judged on the presence of a unit root in each series. Another aspect is based on the presumption that non-stationary spot and futures prices generate a non-stationary hedge ratio (Mallarius and Urrutia, 1991) that challenges a hedging strategy based on perfect and constant hedge ratio. In addition, confirming that the price differences or returns are stationary assist in the decision to apply them in regressions to enable statistical inferences and avoid spurious regressions[7].

Two models are used for this analysis, that is the "augmented Dickey-Fuller test" (ADF) and Phillips and Perron although their results are often not very distinct**.** Three regressions in ADF test are used on each series with the random walk or non-stationary process as a null equation $\Delta y_t = \varpi y_{t-1} + \varepsilon_t$ and a first alternative of a stationary autoregressive process of order 1 (AR(1)); $\Delta y_t = \varpi\, y_{t-1} + \sum^{p}_{i=1} b_i \Delta y_{t-1} + \varepsilon_t$. Lagged differences of individual series are included to ensure that ε_t becomes white noise. The lag parameter (p) is selected using the Akaike information criteria (AIC) and Schwartz Bayesian information criterion (SBIC) based on a maximum lag length of 21. The two information criteria are explained in detail in section 2.2.3.

The second alternative hypothesis includes a constant in the regression; and a constant and trend in third regression. The Phillips Perron runs almost similar regressions but incorporates an automatic correction to the original Dickey-Fuller procedure to allow for serial correlation such that $y_t = \alpha + \beta y_{t-1} + \eta_t$ where η_t is conditioned to be white noise. The unit root test is based on the null hypothesis (H_o: $y_t \sim I(1)$; $\varpi = 1$) and

[7] Regressing non-stationary financial series can violate the assumptions of the distribution of t-ratios and F-statistics (Brooks, 2002; Gujarati, 2003).

alternative hypothesis (H_1: $y_t \sim I(0)$; $\varpi < 1$) with a test statistic[8] that follows a non-standard distribution under the null hypothesis. It is vital to note that critical values of significance differ on the fourth decimal for the Phillips Perron test. In each case, the null hypothesis of a unit root is rejected in favour of the stationary alternative if the test statistic is more negative than the critical value. Other stylised facts tested at various stages include heteroscedasticity and autoregressive conditional heteroscedasticity (ARCH) effects that are transmitted from the logarithmic returns series to the errors of regressions.

2.2.2 Ordinary Least Squares (OLS)

The traditional OLS regression in equation (1) is estimated by regressing the logarithmic spot returns on logarithmic futures returns, represented by ΔS_t and ΔF_t respectively (Ederington, 1979; Figlewski; 1984). The slope ($\hbar_{ols}$) is the constant hedge ratio and it is equivalent to the minimum variance hedge ratio (MVHR).

$$\Delta S_t = c + \hbar_{ols}\Delta F_t + \varepsilon_t \qquad \varepsilon_t \sim i.i.d.\ \mathrm{N}(0, \sigma^2) \quad (1)$$

where; $\Delta S_t = S_t - S_{t-1}$, $\Delta F_t = F_t - F_{t-1}$
ε_t is identically and independently distributed (*i.i.d.*).

The conditions necessary for valid and efficient statistical inference from this model are embodied in assumptions of the classical linear regression model (CLRM) and are investigated using various techniques. It is assumed that the disturbance term (ε_t) in from the regression in equation (1) is normally distributed with zero mean and constant variance or homoscedastic in order to conduct single or joint hypothesis tests about the model parameters. In addition ε_t is assumed to be uncorrelated[9] while ample empirical evidence negates the two assumptions. Numerous authors criticise the minimum-variance hedge ratio from this model based on the inefficiency of the residuals which results.

[8] The test statistics for the original Dickey Fuller tests are defined as t=(ϖ/standard error(ϖ))
[9] It means that the covariance of the residuals of the spot returns and residuals of the futures returns is zero over time.

First, the Jarque-Bera normality test as described before is applied on the joint distribution of the disturbance term (ε_t). The second test named White's (1980) general test without cross-terms in Brooks (2002) is applied as an auxiliary regression of the squared residuals on a constant, the futures returns and squares of the futures returns ($\varepsilon^2_t = \alpha_1 + \alpha_2 \Delta F_t + \alpha_3 \Delta F^2_t + \nu_t$). The null hypothesis is that the coefficients of the regressors except the constant are insignificantly different from zero that is errors are homoscedastic with an alternative hypothesis that errors are heteroscedastic or have a non-constant variance as numerous studies demonstrate. A related effect is the autoregressive conditional heteroscedasticity (ARCH) that is addressed in section 2.2.5. The Breusch-Godfrey test is used to examine the joint null hypothesis that that the current error is not related to any of its r previous values or $\rho_r = 0$, in the auxiliary regression ($\varepsilon_t = c + h_{ols}\Delta F_t + \rho_1\varepsilon_{t-1} + \ldots + \rho_r\varepsilon_{t-r} + \nu_t$) implying no serial correlation in the residuals. The lag order is estimated to be 21 from the average number of trading days that make up the data.

2.2.3 Error Correction Model (ECM)

The omission of the error correction variable from an econometric model when the spot and futures prices are cointegrated causes the estimated hedge ratios and hedging performance to change significantly (Ghosh, 1993; Chou, Denis and Lee, 1996; Lien 1996). Therefore, a hedge ratio is estimated with the ECM based on the two step approach (Engle and Granger, 1987) that incorporates nonstationarity, long-run equilibrium relationship and short-run dynamics and can provide an efficient estimate of the optimal hedge ratio relative to the OLS model. Hence it is imperative to test whether the spot and futures used in this study are integrated and cointegrated before specifying the model.

First, the results from the unit root test on the logarithmic spot and futures prices applied in section 2.2.1 are reviewed in order to ascertain that each series has a unit root or *I(1)*. Then the logarithmic spot prices is regressed on the logarithmic futures prices and a constant ($S_t = \alpha + \beta F_t + u_t$). The saved residuals u_t are also tested for the unit root using equation (2) but the critical values for hypothesis testing are different from those specified for an ADF test on the series of logarithm prices.

Where; H_o is a null hypothesis where the error ($\hat{u}_t$) is nonstationary or *I(1)*. The alternative H_1 means that $\hat{u}_t$ is stationary or *I(0)*. The null is rejected if ϕ is negative and significantly different from zero.

$$u_t = \phi\, u_{t-1} + z_t \qquad\qquad z_t \sim i.i.d.\ \mathrm{N}(0, \sigma^2) \quad (2)$$

$$H_o: \hat{u}_t \sim I(1)$$

$$H_1: \hat{u}_t \sim I(0)$$

Finding that $\hat{u}_t$ is stationary implies that even though the individual series are nonstationary, their linear combination is stationary such that error correction specification must exist. The finding permits the application of the second step. The error correction model is estimated with the spot returns as dependant variable in equation 3. Regressors include a lag of the residuals as an equilibrium error correction term ($\hat{u}_{t-1}$) which eliminates the risk of obtaining a declining and inefficient hedge ratio, lagged values spot and futures returns and a constant. The hedge ratio ($\hbar_{ecm}$) is the slope of the regression estimated as the coefficient of the futures returns.

$$\Delta S_t = \alpha + \hbar_{ecm}\Delta F_t + \sum^{m}_{i=1} \lambda_i \Delta F_{t-i} + \psi \hat{u}_{t-1} + \sum^{n}_{j=1} \theta_j \Delta S_{t-j} + \varepsilon_t \qquad (3)$$

where; $\hat{u}_{t-1} = (S_{t-1} - \alpha - \beta F_{t-1})$.

The optimal lag length of the short-run dynamics (ΔF_{t-i} and ΔS_{t-j}) is required to ensure that ε_t is white noise (Ghosh, 1993). The choice is based on the lag length and or number of parameters that minimises the value of the AIC and SBIC. In estimation, they are represented as follows; [AIC = $ln(\sigma^2) + 2k/T$] and [SBIC = $ln(\sigma^2) + k/TlnT$] where σ^2 is the residual variance, k is the total number of parameters and T is the sample size. The criteria are accorded similar power in this study albeit SBIC imposes a larger penalty for additional coefficients while AIC can deliver two large a model even with an infinite amount of data.

2.2.4 Vector Error Correction Model (VECM)

The VECM in the form of a bivariate vector autoregressive (VAR) process including the actual cointegration relationship prevents an omitted variable bias that prevail in the OLS model and a simple vector autoregressive model (Lien, 1996; Lien, 2004). The steps involved in estimating the VECM are discussed as follows. First, a bivariate VAR as an unrestricted model in equations 4 and 5 is estimated using logarithmic spot and futures prices.

$$S_t = \beta_s + \sum^{l}_{i=1} \varphi_{si} S_{t-i} + \sum^{k}_{j=1} \delta_{sj} F_{t-j} + \varepsilon_{st} \quad (4)$$

$$F_t = \beta_f + \sum^{l}_{i=1} \varphi_{fi} S_{t-i} + \sum^{k}_{j=1} \delta_{fj} F_{t-j} + \varepsilon_{ft} \quad (5)$$

where; $\varepsilon_{s,t}$ and $\varepsilon_{f,t}$ are independently and identically distributed disturbance terms.

The lag length (*l* and *k)* of each series that minimises the multivariate version of AIC and SBIC[10] is selected in order to ensure that $\varepsilon_{s,t}$ and $\varepsilon_{s,t}$ are white noise. The appropriate order of the VAR model facilitates the Johansen (1988) one-step methodology in (Brooks, 2002; Gujarati, 2003) to determine the number of cointegrating vectors for the variables of interest. Two tests are applied simultaneously on the logarithm of each bivariate cash and future series. One is a joint test termed 'trace' statistic (γ_{trace} (r) = $-T \sum^{g}_{i=r+1} ln(1-\lambda_i)$) and the second is the 'maximum eigenvalue' (γ_{max} $(r, r+1)$ = $-T\ ln(1-\lambda_{r+1})$) that conducts separate tests on each eigenvalue. The number of cointegrating vectors under the null hypothesis is r and λ_i is the estimated value for the ith ordered egeinvalue from the long-run coefficient matrix[11]. The full rank of the matrix is equal to g which is equivalent to the number of variables in the system. The decision to reject each null hypothesis is reached if the computed statistic is greater than the critical value at 5 percent significance level.

[10] Information criterion applied over the likelihood ratio (LR) because the former does not require that the errors from each equation be normally distributed.
[11] Specification and discussion available in Brooks, 2002; Gujarati, 2003.

Ultimately, the VECM in equation (6) and (7) has the exact number of cointegration relations built into the specifications such that the error correction term ($\hat{u}_{t-1}$) ensures that the deviation from long-run equilibrium is corrected gradually through a series of partial short-run adjustments.

$$S_t = \alpha_s + \sum\nolimits^{p}_{i=1} \text{¶}_{si}\, S_{t-i} + \sum\nolimits^{q}_{j=1} \text{ЛЬ}_{sj} F_{t-j} - \text{Ю}_s \hat{u}_{t-1} + \pi_{st} \quad (6)$$

$$F_t = \alpha_f + \sum\nolimits^{p}_{i=1} \text{¶}_{fi} S_{t-i} + \sum\nolimits^{q}_{j=1} \text{ЛЬ}_{fj} F_{t-j} - \text{Ю}_f \hat{u}_{t-1} + \pi_{ft} \quad (7)$$

$$\hbar_{vecm} = \rho(\sigma_{s,t} / \sigma_{f,t}) \quad (8)$$

The hedge ratio ($\hbar_{vecm}$) is calculated from the residuals of the model using equation 8. The standard deviations of spot ($\sigma_{s,t}$) and futures ($\sigma_{f,t}$) are obtained from the standard deviation of the errors π_{st} and π_{ft} respectively. The correlation coefficient (ρ) between π_{st} and π_{ft} is obtained from the correlation matrix of the residuals. A larger hedge ratio in value is more efficient than the small hedge ratios obtained from equation 1 and 2 if the case be (Lien 2004).

2.2.5 Generalised Autoregressive Conditional Heteroscedasticity (GARCH) with Error Correction Model

Price volatility demonstrated by time-variation in the distribution of spot and futures returns reflecting the impact of news in the market alongside leptokurtosis influences the joint distribution and properties of a hedge ratio. These factors are not addressed in the OLS estimation which treats a non-constant variance as a problem hence the OLS hedge ratio can be biased and sub-optimal, thereby curtailing its efficiency in reducing the spot price risk (Cecchetti, Cumby and Figlewski, 1988; Myers and Thompson, 1989; Baillie and Myers, 1991; Sephton, 1993). Therefore, the (GARCH) framework by Bollerslev (1986) can offer efficiency gains in estimating hedge ratios. GARCH parametisation has declining weights that never reach zero and offers parsimonious models that are easy to estimate and can predict conditional variances.

A pre-condition for the GARCH model is to provide evidence of ARCH effects in the residuals. This is illustrated in the study of the evolution of the joint distribution or errors which show that the risky times are not scattered randomly across the period but are dependent on the returns, and their variance is autoregressive rendering parameter estimates inefficient. The ARCH effects test (Engle, 1982) using Lagrange Multiplier approach is conducted on the residuals of the OLS model in the regression ($\varepsilon^2_t = \gamma_0 + \gamma_1\varepsilon^2_{t-1} + \ldots + \gamma_q\varepsilon^2_{t-q} + \nu_t$). The test statistic is distributed as chi-square with q degrees of freedom while the joint null hypothesis is that all q lags of the squared residuals have coefficient values that are not significantly different from zero, implying no ARCH effects.

The widely used model in this area is the GARCH (1,1) and the first digit in the parentheses refers to the number of autoregressive lags of the errors or ARCH terms (ε^2_{t-1}), while the second digit is the number of lags of the error variance conditional on news or GARCH terms (σ^2_{t-1}). The order or lag-length of the ARCH and GARCH terms is selected based on the minimisation of the values of AIC and SBIC. Sephton (1993) adopts a univariate GARCH (1,1) model as proposed by Bollerslev (1986) with the first moments allowed in a mean equation as the OLS regression. The choice is based on finding that the spot and futures prices are not cointegrated. However, this study extends the model as prices are assumed to be cointegrated such that the mean equation (9*a*) is an error correction model (Engle and Granger, 1987) similar to the right hand-side of equation (3) in order to prevent downward biasness of the hedge ratio. The constant hedge ratio is the coefficient (h_{garch}) on the futures return.

$$100\,\Delta S_t = \alpha_0 + \theta\Delta S_{t-1} + h_{garch}\Delta F_t + \lambda\Delta F_{t-1} + \psi\hat{u}_{t-1} + \varepsilon_t \tag{9a}$$

$$\varepsilon_t \mid \Omega_{t-1} \sim N(0, \sigma^2_t) \tag{9b}$$

$$\sigma^2_t = \alpha_1 + \beta\varepsilon^2_{t-1} + \gamma\sigma^2_{t-1} \tag{9c}$$

GARCH specification offers convenient assumptions about the conditional density of stock index returns, such as the normal or t distribution in equation (9*b*) leading to a rich model that allows for second-moments in the form of time-dependent conditional variances and leptokurtosis in the unconditional distribution of returns. The error variance (ε_t), conditional on available information at time t – 1 (Ω_{t-1}) is allowed to evolve in a normal distribution based and a conditional variance (σ^2_t). The fitted conditional variance in equation (9*c*) must have a positive value and it is a weighted function of a long-term average value dependent on a constant (α_1), information about volatility during the previous period or arch effect ($\beta\varepsilon^2_{t-1}$) as well as the fitted error variance from the model during the previous period ($\psi\sigma^2_{t-1}$). Effectively, the complete model is an ECM-GARCH (1,1) which assumes that the conditional correlation between cash and futures prices is constant. Parameters α_1 , β and γ are systematically adjusted with the maximum likelihood function, subject to 100 maximum iterations such that a set of parameters values that are most likely to have produced the observed data are chosen.

Chapter 3
Literature Review

3.1 Background

The literature in this section offers a background in hedging with futures while focus is on the hedge ratio estimating techniques to achieve a risk-minimisation objective through the portfolio theory. Considering the evolution of the estimating techniques, it is inevitable to mention application based on commodity and currency futures where necessary albeit emphasis is hedge ratios based stock index futures.

The naïve hedging theory adopts a unit-for-unit hedge ratio with a view that futures markets can prevent a loss arising from price volatility. That is for any given position in the spot market, an investor with a pure risk-minimising objective takes an equal amount of futures contracts to eliminate the price risk of the underlying spot asset such that the difference between the two positions is the negative sign. The major assumption in this theory is perfect correlation whereby the price of the future bought or sold as a hedge increase or decreases by the same amount that the spot price increases or decreases. In this case, the basis is zero and the hedge is perfect (Working, 1953; Johnson, 1960; Ederington, 1979; Hull, 1997; Sutcliffe, 1997).

However, the earliest criticism of the theory recognises that the basis is not stable hence, the objective of a hedge is not to minimise risk, but to maximise profit from the fluctuation of the basis in a speculation manner (Working, 1953). Presumably, an investor adopting the profit-maximisation theory hedges a given long position in the spot market when expecting the basis to decrease and does not hedge when expecting the basis to rise as profit is realised when selling at the high price in future. This view stimulated a debate on the distinction between hedging to reduce risk and speculation in the futures markets. To a certain extent, the argument is endorsed by combining the risk-prevention through naïve positions in the spot and futures markets where one must be the primary market together with a speculative element depending on the investor's reaction expected to non-zero relative price changes (Johnson, 1960). In the latter perspective, the naïve hedge ratio minimises risk only to an investor who expects to receive zero or minimum marginal returns if he undertakes a new risky investment. That is because a bargain with more certain but possibly lower expected return is preferred rather than another bargain with a more uncertain but possibly higher return. Consequently, the expected risk-return function seeks to explain that an investor's optimal position consists of a speculative component and a hedging component.

A modified risk-minimisation perspective developed from the portfolio theory, which is at the centre of this study is superior to both the naïve hedge and profit maximisation approaches. The portfolio approach recognises the existence of basis risk and determines the optimal futures position by minimising the variance of the spot-futures portfolio (Ederington, 1979). The assumption is that futures markets are not biased and the covariance between spot and futures prices is time-invariant. In essence, for any risk-averse investor without consideration of the degree of risk aversion, the optimal hedge ratio, that is the minimum-variance hedge ratio (MVHR) is equal to the unconditional covariance between the spot and futures returns divided by the unconditional variance of the futures return. In a linear regression model constructed with available data, the spot price levels or return is the dependent variable and futures price levels or return is the independent variable, so that the Ordinary Least Square (OLS) estimate of the slope is the estimated optimal hedge ratio. This implies that the hedge ratio is not necessarily 'one' contrary to the naïve hedge ratio and surpasses the portfolio's beta (β) coefficient with respect to the market index because the basis tends to be more volatile in short periods even if dividends are relatively stable. The portfolio theory has another advantage regarding measuring the effectiveness of hedging from an estimated minimum-variance on a portfolio containing futures securities as a fraction of risk on a non-hedged portfolio. It also allows the examination of the effectiveness based on different time horizons.

3.2 Ordinary Least Square Regression

Estimating the minimum-variance hedge ratio using the OLS is approved through empirical evidence in early studies only to generate criticism from the late 1980's. The proportion of output in the case of commodities to be hedged in each contract is obtained from the coefficient of the multiple regression of the cash prices on the futures prices (Anderson and Danthine,1981). However, the focus is on a 'cross hedge' in which case the spot position being hedged is different from the underlying portfolio for the futures contract. In the first empirical analysis in hedging with stock index futures, the hedging performances for the OLS minimum risk hedge ratio is reviewed against the beta hedge ratio (Figlewski, 1984). Application is based on the Standard & Poor's 500 futures contract related to a portfolio underlying the five major stock indexes for the period 1982-83. It is discovered that the minimum risk hedge strategy offers a lower risk and higher return thus dominating the beta hedge that has a higher risk. Including dividend payments in the return series, it is concluded that the dividends do not alter the ex-post minimum variance hedge ratios estimated by OLS using historical data. As a result, a bulk of post-1984 studies on estimating hedge ratios exclude the dividends from stock index and index futures returns.

The assumptions underlying the traditional OLS hedge ratio are generalised under the classical linear regression model (CLRM) thus generating controversy because it does not cater for the unique nature of financial data sufficiently. The major assumption is that asset prices and returns have a normal distribution such that the second moments of the data are constant over time. This is despite the earliest discovery that stock market prices are not normally distributed because high frequency data often contains additional patterns reflecting volatility in the market (Mandelbrot, 1965; Fama 1965) in Officer (1972). Moreover, spot and futures prices are dependent and follow a random walk process which raises possibility for cointegration between two assets. As a result the OLS technique is under scrutiny with alternative techniques being developed to estimate a hedge ratio that is believed to reduce the risk of the spot asset more effectively.

3.3 Generalised Autoregressive Conditional Heteroscedasticity Model

One of the major shortfalls of the OLS technique that renders the estimated hedge ratio unsatisfactory is based on evidence that the distribution of spot and futures returns varies in time, technically bearing time-varying conditional heteroscedasticity. This implies that the covariance and variance in the optimal hedging principle are conditional moments that depend on information available at the time of making the hedging decision (Ceccheti, Cumby and Figlewski, 1988; Myers and Thompson, 1989; Baillie and Myers, 1991). The implication of a hedging strategy based on the constant OLS hedge ratio therefore, is that the joint distribution of spot and futures prices or returns and the conditional covariance matrix are not adjusted to capture their respective changes over time. This can render the OLS hedge ratio incorrect as it can expose the investor to unanticipated basis risk.

An Autoregressive Conditional Heteroscedasticity (ARCH) model envisaged to generate a correct and time-varying hedge ratio is specified containing non-linear constraints (Cecchetti et *al*. 1988). The data used for application comprise 20-year Treasury bonds held for one month in the United States of America. To demonstrate that the joint distributions of the spot and futures are time-variant and the effect of changes in expectations about risk and return, the estimated hedge ratio varies from 0.52 to 0.91. Nevertheless, this work places emphasis on the efficiency of utility maximising optimal hedge rather than variance minimising. In addition, the conditional correlation between spot and futures prices is presumably constant. The preceding literature questions the efficiency of a minimum-variance hedge ratio estimated from the ARCH model and recommends an alternative.

On average, research in futures markets reveals that the generalised autoregressive conditionally heterescedasticity (GARCH) framework by Engle (1982) and Bollerslev (1986) enhances the estimation of the risk-minimising hedge ratio as it fully takes into account the time-varying nature of the second moments of spot and futures returns. In certain applications where a multivariate GARCH framework is used to estimate time-varying hedge ratios it is generalised that it yields superior performance in lowering portfolio volatilities relative to the time-invariant OLS hedge ratio. With the multivariate GARCH, the conditional covariance matrix of the disturbance terms changes systematically over time and estimation is subject to the conditional quasi-maximum likelihood function. Using six commodity futures, the estimated hedge ratio exhibits significant variations when applying the dynamic strategy based on a bivariate GARCH model (Baillie and Myers, 1991). In terms of the ability to reduce risk, this non-constant hedge ratio demonstrates better performance compared with the OLS hedge ratio, suggesting that the latter hedge can be costly for some commodities as indicated in the high variance. The results are based on two versions of the bivariate GARCH namely; the diagonal vectorised conditional covariance (VECH) and the BEKK[12] specifications. Although cointegration does not describe the spot and futures series employed, the conclusion is drawn from evidence showing that the optimal hedge ratios for each of the six commodities are non-stationary. This view is considerable even in cross-hedging where the hedge ratio has to be adjusted regularly in order to reflect the different dimensions of basis risk (Castelino, Francis and Wolf, 1991).

Both the univariate and multivariate GARCH models used to estimate time-varying hedge ratios for three commodities traded on the Winnipeg Commodity Exchange (Sephton, 1993) are compared with the OLS. While the emphasis is on the risk-return trade-off again, it is reported that the multivariate GARCH model is best in solving for the temporal evolution in the processes generating the spot and futures prices. As such the hedge ratio from the GARCH model leads to a lower conditional variance of market returns and efficiency gains than the OLS despite that the estimated hedge ratios are stationary and the prices are not cointegrated.

The bivariate VEC-GARCH model with the BEKK parameterisation is also estimated to display the asymmetric effect of news on hedging using 3850 daily observations of the FTSE-100 stock index and stock index futures contract spanning the period January 1985 to April 1999 (Brooks, et al. 2002). The research shows that asymmetric models produce more effective hedge ratios because they allow positive and negative price innovations to affect volatility forecast in different ways. In extending the same study to investigate hedging performance, it is discovered that the time-varying hedge

[12] BEKK represents researchers Baba, Engle, Kraft and Kruner whose work was consolidated to form this simple model (Engle and Kroner, 1995) in Park and Switzer (1995).

results in a considerable improvement yet allowing for asymmetries results in only a very modest incremental reduction in the hedged portfolio risk.

The constraint for a positive-definite conditional variance-covariance matrix of the spot and futures returns for all values of the errors in a sample in the bivariate GARCH (1, 1) in VECH forms as employed by Baillie and Myers (1991); Brooks, et al. (2002) may be too complex to some hedging practitioners. Furthermore, the matrix has an excessive number of parameters that should be subject to inequality restrictions as well as nonlinear optimisation algorithms. As such, a restricted version of the bivariate BEKK in the form of a bivariate co-integrating model, with GARCH error structure (BGARCH) is employed by Park and Switzer (1995) as well as Floros and Vougas (2004). The BEKK parameterization has the advantages of requiring the estimation of only 11 parameters in the conditional variance-covariance structure and of guaranteeing that is positive definite.

Estimates of the risk-minimising future hedge ratios for the daily Standard and Poor's 500 Index Futures, Major Market Index futures and the Toronto 35 Index Futures are obtained from the OLS and the bivariate GARCH (Park et al. 1995). While the GARCH hedging method provides the optimal hedge ratio, caution is raised about the trade-off between the risk reduction and transaction costs that should determine the extent of practical application of GARCH in risk management. Floros and Vougas (2004) estimate the mean hedge ratios from the restricted versions of bivariate BEKK (BGARCH (1,1)) model and compare with the constant hedge ratios from the OLS model, ECM and VECM. The conclusion drawn is that the large mean hedge ratio from the BGARCH model should be more efficient in reducing the risk of spot prices which in turns implies that the other three hedge ratios underestimate the number of futures contracts required to hedge the risk. In addition, the hedge ratios obtained from the BGARCH (1,1) are time-varying reflecting the arrival of new information in the Greek market.

Even though recommendation of the GARCH model on additional evidence of non-stationary hedge ratios and hedging effectiveness that is also non-stationary based on the Standard & Poor's 500 index, New York Stock Exchange index and four foreign currencies (Malliurus and Urrutia, 1991) caution is raised pertaining the transaction costs that may be associated with dynamic hedging. Thus, a suggestion for an investor is to conduct a cost-benefit analysis to ascertain the potential risk reduction. In terms of performance of the various model specifications, optimal hedge ratios also prove to be non-constant reflecting significant theoretical advantages of the GARCH models, however these models perform only slightly better in risk reduction than a simple constant hedge ratio (Myers, 1991). For this reason, Myers asserts that estimating constant optimal hedge ratios and using linear regression approaches, may be an acceptable approximation. Lien and Luo (1994) also dispute the supposed superiority of GARCH to all the other models in the hedge ratio estimation. The premise is that the cointegrating relationship between the prices is the most critical factor for consideration in order to optimise the hedging performance of the obtained ratio. Following suit, a comparison of the performances of minimum variance hedge ratio from the OLS model and time-varying hedge ratio from the constant correlation (CC) GARCH which alleviates the problem of structural changes (Lien et al. 2002) is presented based on out-of-sample forecasts. It is discovered that the bivariate GARCH model incurs 20 percent more risk although the magnitude of the difference may not be large for some markets. This finding supports the argument that albeit conditional heteroscedasticity is a characteristic of numerous financial series there is no definitive conclusion about the superior hedging performances of the multivariate GARCH model. Observations used in the estimation of the OLS and CC-GARCH models include ten pairs of spot and futures series covering currency, commodity and stock index futures while some of the bivariate series violate the constant correlation assumption. Highlighting the high computational costs of the GARCH estimation, the authors argue that GARCH specification should not be adopted for practical hedging purposes yet it is useful for data description.

3.4 Error Correction Model

Another approach developed to solve some of the problems inherent in the traditional OLS model considers cointegration and thereby estimates the long-term equilibrium relationship between spot and futures prices or returns yet allows for short-run deviations in the hedge ratio attributable to basis risk. Engle and Granger (1987) show that if two or more series are cointegrated, then there exists an error correction model linking the series. The theoretical cointegration relationship between the price of a stock index futures contract and the price level of the underlying index is the cost-of-carry[13] hypothesis. In efficient and continuous spot and futures markets where transaction costs are absent, risk-less arbitrage profit opportunities should not appear so the cost-of-carry relation should be satisfied at every time during the life of the futures contract (Stoll and Whaley, 1990). When transaction costs are present, the relationship is not perfect, but holds within arbitrage bounds around the true relationship.

The existence of a cointegrating relationship between spot and future markets implies that an error correction term and past values of spot and futures are essential in model specification otherwise the hedge ratio is downwardly biased (Ghosh, 1993). Applying Engle and Granger's two-step approach in empirical estimation based on several stock portfolios hedged with Standard & Poor's 500 index futures, Ghosh finds that hedge ratios obtained from OLS models to be under-estimated. The large hedge ratios obtained from the ECM are attributed to the correction for the misspecification of the OLS as evidenced by significant likelihood ratio tests. In addition to the basis risk and correlation between the spot and futures prices, another factor identified as a determinant of hedge construction and its effectiveness is the hedging horizon. In accordance, Lien and Luo (1993) confirm the cointegrating relationship in empirical analysis and recommend an error correction model to estimate the hedge ratios for stock index markets. An investor makes a mistake if the hedging decision is based on the hedge ratio obtained from a first difference model that does not include an error correction term (Lien, 1996).

[13] This notion implies that a non-stationary basis (F_t-S_t) offers an opportunity for risk-less arbitrage profits to traders although eroded quickly as the prices get restored to equilibrium.

The previous overview on the statistical power of the ECM in estimating a risk-minimising hedge ratio is confirmed in studying the Nikkei Stock Average (NSA) index and NSA index futures (Chou, Denis and Lee, 1996). Again, the likelihood ratio statistic as well as the Lagrange Multiplier test for the ARCH effects test reveals that the error correction model is a better specification than the OLS. In addition, temporal aggregation affects the magnitude of the estimated hedge ratios. In a slight extension, a fractionally integrated error correction (FIEC) is used for estimation and compared with the simple ECM, OLS and vector autoregressive (VAR) models using daily data for the Nikkei Stock Average Index over the period 1989 to 1996 (Lien and Tse, 1999). Results show that the ECM provides a consistently larger hedge ratio, with highest return and lowest risk than the FIEC, VAR and OLS. Therefore, incorporating the fractional cointegration relationship does not improve the hedging performance over the ECM. In the same study, the OLS provides the smallest and worst outcomes for hedging periods longer than 5 days compared with the other methods.

Floros and Vougas (2004) also utilise FTSE/ASE-20 stock index and stock index futures contract for the period January 2000 to August 2001 as well as the FTSE/ASE Mid 40 stock index and stock index futures contract for the period January 2000 to August 2001 to compare the OLS and ECM models. While the ECM estimates a large hedge ratio for FTSE/ASE-40, a small hedge ratio is generated from the ECM for FTSE/ASE-20. The analysis presents that even the smaller hedge ratio from the ECM is more efficient as few contracts are required to hedge the spot portfolio while the OLS ratio overestimates it the contracts.

In an attempt to address serial correlation that can cause bias in the optimal hedge ratio a bivariate vector autoregressive (VAR) model is estimated and its performance is compared with the OLS, ECM and the multivariate diagonal VECH-GARCH models using the All Ordinaries Share Price Index (AOI) and the corresponding index futures contract (SPI) in the Australian market (Yang, 2001). Nevertheless, the results indicate that the error correction model still provides the best estimate of the hedge ratio and reduces risk more effectively than all three alternative models. In this case, the findings emphasise the importance of the cointegration relationship between spot and futures markets. In addition, the inability of the ordinary VAR to improve the estimation confirms the lack of a theoretical basis in futures hedging yet an extension to include the error correction term in the form of a vector error correction model (VECM) is likely to yield better performance (Lien, 2004). While in agreement with a cointegrating relationship between spot and futures market, the univariate and multivariate error correction models do not result in economically significant risk-reduction for debt contracts in New Zealand and Australian markets thus (Wilkinson, Rose and Young, 1999). This difference is associated with the non-synchronous trading in the markets as well as negative term structures.

In conclusion, the literature in futures markets covers the major characteristics of spot and futures prices for consideration in model specification designed to estimate optimal hedge ratios that are ignored in the traditional ordinary least square model by Ederington (1979). Since the generalised autoregressive conditional heteroscedasticity model enables the modelling of the dependent and time-varying second moments of the data the statistical performance of the GARCH model in risk-reduction tends to be superior as it corrects for inefficiency and downward biasness of the estimated hedge ratio. Empirical evidence suggests that the multivariate GARCH model also enables the continuous review of the hedge ratio in order to avoid potential losses from unanticipated basis risk. With particular focus in the long-run equilibrium relationship between the prices, on average the error correction model and to a less extent the vector error model generate better estimates than the OLS. However, in a few cases empirical results are contrary to the preceding generalisation prompting a recommendation for detailed data diagnosis in any spot and futures market before adopting a hedging technique for reducing risk in a portfolio of assets.

Chapter 4
Empirical Evidence

4.1 Data

This study employs 1043 daily observations after removing non-trading days on the FTSE/JSE Top 40 stock index and stock index futures contract for the period 2nd January 2002 to 28th February 2006. The sample size is large relative to other studies and this is expected to curb inflated standard errors. Closing prices for the spot index were obtained from DataStream data set while closing futures prices were obtained from the official webpage of the South African Futures the FTSE/JSE Exchange (http://www.safex.co.za). FTSE/JSE Top 40 stock index consists of the largest 40 companies ranked by full market capitalisation (value) that is before the application of any weightings in the All Share Index. Market capitalisation is the market value of a company's outstanding shares obtained by multiplying the stock price by the total number of outstanding shares. The futures contract is the FTSE/JSE's Top 40 future nearest to expiration, assuming a rollover to the next contract expiration. Analysis is confined to the nearby contract because almost all trading volume is in the near month so that liquidity is much great in that contract compared with the far contract. The futures contracts are quoted in the same units (South African Rand) as the underlying index without decimals, with the price of a futures contract or contract size being the quoted number (index level) multiplied by the contract multiplier, which is R10 for the contract. Futures expiry months are March, June, September and December in this market. The index futures contract is cash-settled and marked to market[14] on the last trading day, that is at 16:00 South African time on the third Thursday (or previous day if that Thursday is a public holiday) in the delivery or expiration month. The formal futures exchange was established in 1988 as well as the SAFEX clearing company. It is licensed in terms of the Financial Markets Control Act of 1989 to trade in futures and in options on futures.

[14] Marking to market means making daily adjustment of futures contract prices to reflect profit or loss. The open positions are marked to market to the cash price prevailing on the close of the futures trading and the gains and losses are debited or credited for individual member accounts.

4.2 Evidence on Stylised Facts

The characteristics of the financial series are presented in section 4.2 and these are incorporated in the various econometric modelling techniques for estimation of the optimal hedge ratio in subsequent sections.

4.2.1 Graphical Analysis

First, the logarithm of stock index (spot) and stock index futures prices in Figure 1 and 2 respectively tend to explode away from the mean of the series while the mean appears to be changing over time. The line graphs suggest that the both prices have a long memory and follow a random walk or non-stationary process. A cointegrating relationship between the prices is likely to exist as the figures are somewhat similar over the sample period. Results of the statistical tests are presented in a subsequent section.

Figure 1: Logarithmic Stock Index

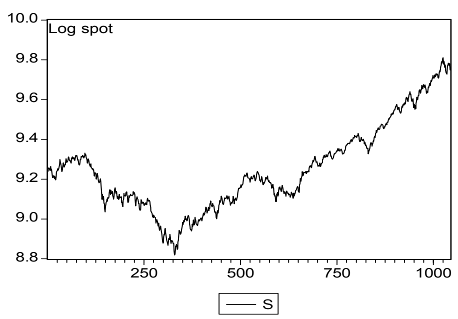
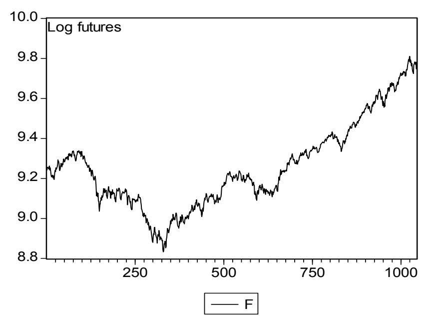


Figure 2: Logarithmic Index Futures



Second, returns generated as the first-difference of the logarithmic spot and futures prices are presented in Figures 3 and 4, illustrating a considerably different pattern. There are signs of a mean-reverting process on the series which is in an indication that the returns are weakly-stationary. Mean reversion is a tendency for a stochastic process to remain near, or return to a long-run average over time such that if a stock or futures contract is underperforming, its price will move towards its average value when the market rebounds. Volatility 'clustering' or 'pooling' (Pagan, 1996; Engle, 2001; Brooks, 2002) is also evident on the graphs as the amplitude of the daily returns varies over time. This means that the current level of volatility tends to be positively correlated with its level during the immediately preceding periods and influenced by news. It is also a pointer to autoregressive conditional heteroscedasticity (ARCH) effects in the returns. The clustering cause uncertainty whose costs may be reduced with a continuously adjusted hedge ratio estimated with a dynamic or multivariate GARCH model.

Figure 3: Stock Index Returns

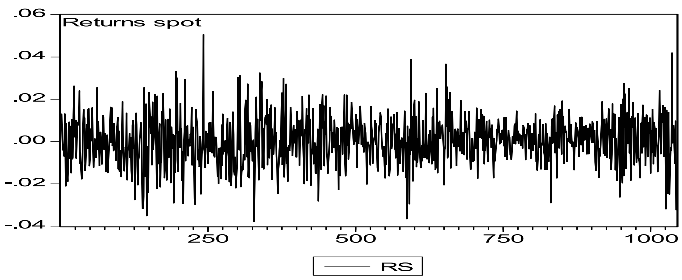
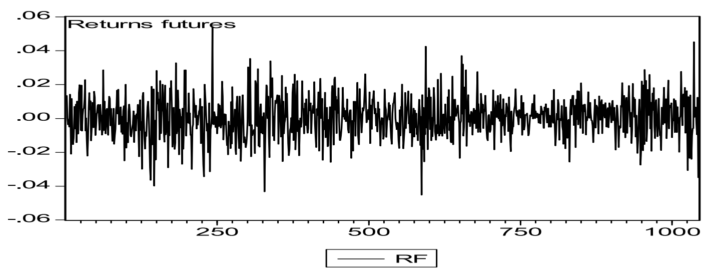


Figure 4: Index Futures Returns



4.2.2 Unit Root Test

The results of Augmented Dickey-Fuller (ADF) and Phillips Perron unit root tests with computed test-statistic and tabulated critical values from Engle and Granger (1987) are consolidated in Table 1. The null hypothesis is that data follows a random walk process or has a unit root. Results presented reflect only alternative of a stationary autoregressive process of order one. Other alternatives of a stationary with drift as well as stationary with drift and a deterministic time trend are tested but results are not presented because they lead to similar conclusions.

Similarly, the test statistics are more or less the same whether the lag length is one that optimises the AIC or SBIC. On one hand, the test statistics are positive and greater than the critical values for the logarithmic spot (*S*) index and index futures (*F*) under both the ADF and PP tests. The null hypothesis is not rejected because of the evidence of a unit root at 5 percent significance level. This implies that the series are nonstationary and the finding is consistent with weak-form efficiency of the cash and futures markets. On the other, the application of the ADF and PP on the first difference series; ΔS and ΔF provides evidence to reject the null hypothesis of the presence of a unit root at 5 percent significance level. This is because the test statistics are more negative than the critical values. Therefore, the first-difference series are stationary. Financial theory suggests, returns on both the stock index and index futures are weakly stationary (Brooks, 2002). A conclusion is drawn that each series is an I(1) process which is necessary for testing the existence of cointegration.

Table 1: Unit Root Tests

		Augmented Dickey Fuller			Phillips Perron		
	Significance level	Critical value	Test statistic	Decision	Critical Value	Test statistic	Decision
S	1%	-3.4345	1.3826	I(1)	-3.4345	1.4116	I(1)
	5%	-2.8632	1.3826	I(1)	-2.8632	1.4116	I(1)
	10%	-2.5677	1.3826	I(1)	-2.5677	1.4116	I(1)
F	1%	-3.4345	1.3240	I(1)	-3.4345	1.3812	I(1)
	5%	-2.8632	1.3240	I(1)	-2.8632	1.3812	I(1)
	10%	-2.5677	1.3240	I(1)	-2.5677	1.3812	I(1)
ΔS	1%	-3.4345	-19.5447	I(0)	-3.4345	-30.1362	I(0)
	5%	-2.8632	-19.5447	I(0)	-2.8632	-30.1362	I(0)
	10%	-2.5677	-19.5447	I(0)	-2.5677	-30.1362	I(0)
ΔF	1%	-3.4345	-31.0158	I(0)	-3.4345	-30.9923	I(0)
	5%	-2.8632	-31.0158	I(0)	-2.8632	-30.9923	I(0)
	10%	-2.5677	-31.0158	I(0)	-2.5681	-30.9923	I(0)

$H_o: \Delta y_t = \varpi y_{t-1} + \varepsilon_t : \varpi = 1;$

$H_1: \Delta y_t = \varpi y_{t-1} + \sum^{p}_{i=1} b_i \Delta y_{t-1} + \varepsilon_t : \varpi < 1$

4.2.3 Distribution

The Jarque Bera statistic for the unconditional distributions of series confirms the general characteristic distribution of asset prices as it provides no support for a null hypothesis of normal or mesokurtic distribution assumption, hence the null hypothesis is rejected at 5 percent significance level (Table 2). Beginning with the unconditional distributions of logarithmic spot and futures prices, the mean is non-zero in both cases while the standard deviation suggests that the spot prices are slightly more volatile than the futures. Positive skewness persists, as the coefficient is greater than zero that is usual with daily trading prices. A platykurtic character that is an almost flat top around the mean of both prices is expected and this is supported by the kurtosis that is less than three.

Table 2: Distribution of Prices, Returns and OLS Residuals

	S	F	ΔS	ΔF	ε_t
Jarque Bera	67.4044	67.9252	23.7604	6596.33	9905.31
Probability value	0.0000	0.0000	0.0000	0.0000	0.0000
Mean	9.2494	9.2549	0.00049	0.00048	3.84E-20
Standard deviation	0.2167	0.2151	0.0114	0.0119	0.003594
Skewness	0.6152	0.6176	0.0431	0.0191	0.532262
Kurtosis	2.8071	2.8068	3.7347	3.9514	18.06692

Null hypothesis: S, F, ΔS, ΔF, ε_t individually ~ $N(0, \sigma^2)$.

Similarly, the mean of the spot and futures returns is non-zero as expected. Excess kurtosis (coefficient of kurtosis minus 3) is concluded that the returns have a distinct peak near the mean, declining rather rapidly with fat tails. This slightly leptokurtic characteristic is in accordance with numerous studies in the subject where the futures returns have more intense peakedness around the mean. Spot and future returns also have long right tails relative to the left tail which is a slight difference from negative skewness that generally characterises stock returns as time interval increases. Chou et. *al* (1996) also found that the daily returns on the Nikkei spot and futures are skewed to the right relative to the one to five weeks returns. The results support Baillie at al. (1991) as a justification for the use of a GARCH model to estimate the hedge ratio. Research index futures markets exhibits more volatility in futures returns relative to spot returns, but only slight difference between the standard deviations of 0.0119 for futures and 0.0114 for the returns is observed in this study. As mentioned earlier, the mis-pricing of the futures contract increases its volatility.

4.3 Ordinary Least Square

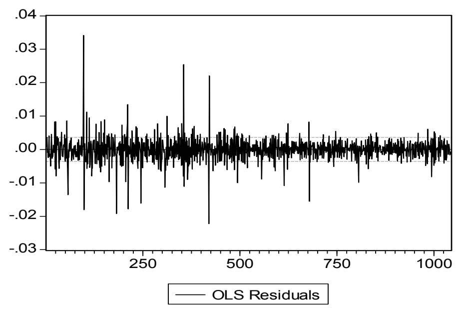
The estimation of the hedge ratio by the regressing of daily spot returns on the futures returns using the OLS technique (equation 1) is applied. Before the discussing the hedge ratio in Table 3, it is necessary to examine the empirical findings from the model regarding the assumptions of the classical linear regression model. The normality assumption for the disturbance term (ε_t) does not hold as the Jarque-Bera statistic (9905.31) in Table 2 is significantly different from zero leading to the rejection of the null hypothesis at 5 percent significance level. In addition, ε_t is positively skewed with an excess kurtosis of 15. Clearly the fat-tails of the ε_t are generated from the fat-tails of the returns, justifying the study of the distribution of the unconditional distributions in section (4.2.3). Table 3 shows results from the White's test statistic using the Lagrange Multiplier approach, from which the joint null hypothesis that the errors are homoscedastic (where $\alpha_2 = 0$, $\alpha_3 = 0$) and independent of the futures returns is rejected at 5 percent level. This is because the White's test statistic computed as the product of R^2 from the computerised auxiliary regression and the number of observations (T) is more than the critical value of the chi-square (χ^2) at 2 degrees of freedom.

Table 3: White's Heteroskedasticity Test			
F-statistic	2776.231	Probability	0.00000
Obs*R-squared	877.7515	Probability	0.00000
R^2	0.8423		
Included observations (T)	1042		

Test: $\varepsilon^2_t = \alpha_1 + \alpha_2 \Delta F_t + \alpha_3 \Delta F^2_t + \nu_t$

Therefore, the squared disturbance term is characterised with a roughly constant mean value while the variance changes systematically with the return series, a process termed dependent heteroscedasticity. Figure 4 in section 4.2.1 and Figure 5 follow the same pattern, demonstrating the heteroscedasticity and ARCH effects.

Figure 5: OLS Residuals



The joint null hypothesis of no serial correlation in the errors based on the Breusch-Godfrey test using the Lagrange Multiplier again (Table 4) is rejected at 5 percent level. The decision is based on the test statistic computed as (T-r) multiplied by R^2 which clearly exceeds the critical value from the χ^2 table at 21 degrees of freedom. The results also support the assumption that errors suffer from autoregressive conditional heteroscedasticity (ARCH).

Table 4: Breusch-Godfrey Serial Correlation LM Test

F-statistic	8.2219141	Probability	0.00000
Obs*R-squared	150.9763	Probability	0.00000
R^2	0.144891		
Included observations (T-r)	1022		

Test: $\varepsilon_t = c + \hbar_{ols}\Delta F_t + \rho_1\varepsilon_{t-1} + \ldots + \rho_{21}\varepsilon_{t-21} + v_t$

The preceding tests influence the conclusion drawn about the estimated constant minimum-variance hedge ratio ($\hbar_{ols}$) presented in Table 5 which is significantly different from zero at 5 percent level. The ratio is positive and less than one ($\hbar_{ols} < 1$) suggesting that the spot and futures market are not perfectly correlated hence the use of the naïve or unit-for-unit hedge ratio can be costly and ineffective in reducing the exposure spot price risk as established in the modern portfolio hedging theory.

Table 5: Ordinary Least Square				
	Coefficient	Standard error	t-ratio	Probability Value
c	4.92E-05	0.000111	0.441402	0.6590
ΔF_t	0.905703	0.009347	96.89920	0.0000*

Model 1: $\Delta S_t = c + h_{ols}\Delta F_t + \varepsilon_t$
*Notes: * indicates significance at 5% level.*

The variance of the errors is positively related to the square of the futures returns causing a very low standard error for the intercept thereby confirming that the linear specification of the model is misleading. One approach to eliminating the inefficiency of h_{ols} caused by the dependent and time-varying errors is using the GARCH model, which generates heteroscedasticity-consistent standard error estimates often termed 'robust'.

4.4 Error Correction Model

This section presents results from the ECM, which presumes that the OLS is misspecified because of the omission of the error correction term that has an effect on the spot price behaviour. First, it is ensured that the logarithmic spot price and futures price are I(1). Table 1 in section 4.2.2 does not provide a reason to reject the null hypothesis of a unit root at 1, 5 and 10 percent significance levels and the prices are I(1). Statistical inference is not made from the coefficients of the parameters in the estimated cointegrating equation ($S_t = \alpha + \beta F_t + u_t$) in this case because of spurious results as the series are non-stationary and there are no short-run dynamics. The residuals u_t are tested with the ADF and PP for the unit root in a similar procedure as before and are found to be I(0) or stationary as presented in Table 6. The null hypothesis of unit root is rejected at 1, 5 and 10 percent significance level. Therefore, there is evidence of cointegration between the spot and futures prices although the hypothesis testing concerning the cointegrating relationship is presented at later stage.

Table 6: Unit Root Test on Residuals							
		Augmented Dickey Fuller			Phillips Perron		
	Significance level	Critical value	Test statistic	Decision	Critical value	Test statistic	Decision
u_t	1%	-4.00	-25.28364	I(0)	-4.00	-52.48652	I(0)
	5%	-3.37	-25.28364	I(0)	-3.37	-52.48652	I(0)
	10%	-3.02	-25.28364	I(0)	-3.02	-52.48652	I(0)

The cointegration effect is therefore applied as one of the explanatory variables in the second step which is estimating the ECM (equation 3) with the least square method. The model reflects that the change in spot price series is not only a function of the change in futures price series, but also a function lagged equilibrium error ($\hat{\boldsymbol{u}}_{\mathbf{t-1}}$) and lagged values of the changes in spot and futures price series. The optimal lag length for the short-run dynamics is three based on the AIC and SBIC in Table 7.

Table 7: ECM Lag-length Selection		
Lag length ($\Delta S, \Delta F$)	Akaike	Schwarz
(0,0)	-8.513079	-8.498831
(1,1)	-8.579297	-8.555532
(2,2)	-8.590619	-8.557322
(3,3)	-8.612550*	-8.569707*
(4,4)	-8.608683	-8.556279
(5,5)	-8.604562	-8.542582
(6,6)	-8.606754	-8.535184
(7,7)	-8.603576	-8.522401
(8,8)	-8.599162	-8.508367
(9,9)	-8.597320	-8.496890
(10,10)	-8.595376	-8.485296

*Notes: * Lag length that minimises information criterion.*

Table 8 presents results from which an improved optimal hedge ratio (h_{ecm}) is contrasted with the one estimated in equation 1 (h_{ols}). Coefficient estimates of the ECM are asymptotically efficient except the estimate of the cointegrating parameter. The co-integrating hedge ratio ($\hat{h}_{\mathrm{ecm}}$) is 0.9199 and less than one. It is significantly different from zero at 5 percent significance level and slightly large compared with the OLS hedge ratio ($\hat{h}_{ols}$). The results demonstrate that the OLS hedge ratio is inefficient and underestimated, thereby resulting in a smaller than optimal hedge of the spot position or portfolio (Ghosh, 1993). Investors require more futures contracts as estimated by the ECM to attain the minimum-variance in the spot position.

Table 8: Error Correction Model

	Coefficient	Standard error	t-ratio	Probability value
α	4.38E-05	0.000101	0.432858	0.6652
ΔF_t	0.919907	0.008643	106.4286	0.0000*
F_{t-1}	0.325896	0.032069	10.16228	0.0000*
F_{t-2}	0.160698	0.032394	4.960772	0.0000*
F_{t-3}	0.104594	0.029844	3.504750	0.0005*
$\hat{u}_{t-1}$	-0.106389	0.020003	-5.318678	0.0000*
ΔS_{t-1}	-0.305613	0.033302	-9.177099	0.0000*
ΔS_{t-2}	-0.157887	0.033722	-4.681971	0.0000*
ΔS_{t-3}	-0.138996	0.030920	-4.495323	0.0000*

Equation 3: $\Delta S_t = \alpha + h_{ecm}\Delta F_t + \sum^{m}_{i=1} \lambda_i \Delta F_{t-i} + \psi\hat{u}_{t-1} + \sum^{n}_{j=1} \theta_j \Delta S_{t-j} + \varepsilon_t$

*Notes: * indicates significance at 5% level with a critical value is 1.6649.*

The error correction term ($\hat{u}_{t-1}$) representing the long-run equilibrium relationship is negative and significantly different from zero at 5 percent level. The negative sign indicates that if the difference between the logarithmic spot and futures prices is positive in one period, the spot price will fall during the next period to restore equilibrium. Moreover, short-run deviations from the hedge ratio deduced from (λ_i and θ_j) arising from a non-constant basis risk have a significant impact on the optimal hedge ratio because they are significantly different from zero at 5 percent level.

4.5 Vector Error Correction Model

The use of the VECM to estimate an optimal hedge ratio is preceded by the estimation of the unrestricted vector autoregressive model (equations 4 and 5) to allow the selection of the appropriate order. Table 9 shows that the multivariate AIC and SBIC are minimised at the fourth lag-length of each series.

Table 9: VAR Lag-length Selection

Lag length	Akaike	Schwarz
1	-14.54919	-14.52069
2	-14.61328	-14.56574
3	-14.63893	-14.57234
4	-14.66109*	-14.57541*
5	-14.65274	-14.54793
6	-14.64527	-14.52131
7	-14.64618	-14.50304
8	-14.63904	-14.47668
9	-14.63426	-14.45267
10	-14.62919	-14.42833

*Notes: * Lag length that minimises information criterion.*

Results of the VAR of order 4, with lagged values of the logarithmic spot and futures prices are shown in Table 10. The asterisk (*) indicates the coefficients that are significantly different from zero at 5 percent level. The VAR(4) is used as a base to test hypothesis about the cointegrating vectors using Johansen's test.

Table 10: Vector Autoregressive Model

	S			F		
	Coefficient	Standard errors	t-ratio	Coefficient	Standard errors	t-ratio
β	-0.004252	0.01586	-0.26804	0.005348	0.01651	0.32393
S(-1)	0.979155	0.10647	9.19672*	0.425297	0.11082	3.83784*
S(-2)	0.433971	0.12403	3.49886*	0.311195	0.12910	2.41053*
S(-3)	-0.463610	0.12370	-3.74780*	-0.524431	0.12875	-4.07309*
S(-4)	0.089332	0.10257	0.87095	0.626421	0.10676	5.86769*
F(-1)	-0.485112	0.11621	-4.17453*	-0.347811	0.12095	-2.87555*
F(-2)	0.349122	0.11711	2.98110*	0.440460	0.12190	3.61342*
F(-3)	-0.048422	0.10333	-0.46862	0.060953	0.10755	0.56674
F(-4						

Equation 6: $S_t = \beta_s + \sum_{i=1}^{t} \varphi_{si} S_{t-i} + \sum_{j=1}^{k} \delta_{sj} F_{t-j} + \varepsilon_{st}$

Equation 7: $F_t = \beta_f + \sum_{i=1}^{t} \varphi_{fi} S_{t-i} + \sum_{j=1}^{k} \delta_{fj} F_{t-j} + \varepsilon_{ft}$

The logarithmic spot index and index futures prices that have been proven to be I(1) have only one cointegrating vector. Evidence is presented from the Johansen unrestricted cointegration test in Table 11. The assumption in the model is that there is an intercept but no trend in VAR and the cointegrating vector. The null of no cointegrating vector is rejected at 5 percent significance level because the trace statistic (maximum eigenvalue), of 35.57 (34.85) is greater than the 5 percent critical value of 15.49 (14.26). The opposite holds for the second null hypothesis of one cointegrating equation, as the trace and maximum eigenvalue statistics are 0.71 and less than the critical value of 3.84 therefore the null is not rejected.

Table 11: Johansen - Cointegration Test		
Rank Test (Trace)		
Cointegrating vectors under null hypothesis	Trace statistic	Critical value
r = 0*	35.56604	15.41
r = 1	0.712831	3.76
Rank Test (Maximum Eigenvalue)		
Cointegrating vectors under null hypothesis	Max-Eigen statistic	Critical value
r = 0*	34.85321	14.07
r = 1	0.712831	3.76

*Notes: * denotes rejection at 5 percent level. Critical values from Osterwald-Lenum (1992) in Brooks (2002).*

The results mean that only one linear combination of the cointegrating vector is stationary. Moreover, the assumption of a long-run equilibrium relationship between the prices which embodied in the cost-of carry theory is valid in this case. Subsequently, results of the VECM (equations 6 and 7) from the VAR(4) with error correction term are presented in Table 12. ΔS and ΔF represent the differenced logarithmic spot and futures prices at each lag. It is shown that unlike in the ECM, the coefficients (ƗO_s and ƗO_f) of the error-correction terms are positive confirming that the residuals of the cointegrating equation are positive. Moreover, ƗO_f is large in value and significantly different from zero at 5 percent level, implying that the futures price series has a great speed of adjustment to the previous period's deviation from long-run equilibrium than the spot price series.

Table 12: Vector Error Correction Model

Error Correction	ΔS			Δ F		
	Coefficient	Standard errors	t-ratio	Coefficient	Standard Errors	t-ratio
α	0.000492	0.00035	1.40282	0.000481	0.00037	1.31840
$\hat{u}_{t-1}$	0.096091	0.07028	1.36718	0.217480	0.07316	2.97280*
ΔS(-1)	-0.124300	0.11886	-1.04578	0.205224	0.12372	1.65884*
ΔS(-2)	0.315916	0.12007	2.63106*	0.522198	0.12498	4.17829*
ΔS(-3)	-0.142609	0.11814	-0.14260	0.004429	0.12296	0.03602
ΔS(-4)	-0.018779	0.10823	-0.17351	0.006180	0.11265	0.05486
ΔF(-1)	0.192181	0.11438	1.68026*	-0.152638	0.11905	-1.28214
ΔF(-2)	-0.296890	0.11665	-2.54515*	-0.505296	0.12142	-4.16169*
ΔF(-3)	0.048144	0.11363	0.42369	-0.069601	0.11827	-0.58848
ΔF(-4)	0.000353	0.10407	0.00339	-0.018553	0.10832	-0.17127
Cointegrating Equation	**Coefficient**	**Standard errors**	**t-ratio**			
S(-1)	1.0000000	-	-			
F(-1)	-1.007187	0.00395	-254.683			
α	0.072059	-	-			

Equation 8: $S_t = \alpha_s + \Sigma^p{}_{i=1}¶_{si} S_{t-i} + \Sigma^q{}_{j=1}Љ_{sj}F_{t-j} - Ю_s\hat{u}_{t-1} + \pi_{st}$

9: $F_t = \alpha_f + \Sigma^p{}_{i=1}¶_{fi}S_{t-i} + \Sigma^q{}_{j=1}Љ_{fj}F_{t-j} - Ю_f\hat{u}_{t-1} + \pi_{ft}$

The estimation of the hedge ratio from the VECM is based on two standard deviations; of spot ($\sigma_{s,t}$) as the standard deviation of π_{st} (0.0112) and futures price ($\sigma_{f,t}$) as standard deviation of π_{ft} (0.0116) generated from the residuals in Table 13. Moreover, the correlation coefficient (ρ) between π_{st} and π_{ft} is 0.957 obtained from the correlation matrix of the residuals. Despite improved specification in terms of a complete cointegrating system, the hedge ratio (h_{vecm}) is 0.9199 and equivalent to the ECM hedge ratio. Therefore, all other things remaining equal, an investor can effectively use the ECM over the VECM to hedge the risk exposure in the spot position when given these two options only. Yang (2001) also discovers that the ECM provides a hedge ratio that is large in value relative to the VAR model. The non-normal and leptokurtic behaviour of the individual error terms is also observed from this model and consistent with earlier findings.

Table 13: Vector Error Correction Model – Residuals		
	Spot(π_{st})	Futures (π_{ft})
Mean	1.07E-19	-1.06E-18
Standard Deviation	0.011212	0.011670
Skewness	0.021045	-0.020971
Kurtosis	3.663651	3.787741
Jarque-Bera	19.12533	26.91424
Probability	0.000000	0.000000
Correlation coefficient (ρ)	0.95744	
$h_{\text{vecm}} = \rho(\sigma_{s,t}/\sigma_{f,t})$	0.9199	

4.6 GARCH (1,1) with Error Correction Model

The appropriateness of the GARCH model for the spot and futures returns is judged by the presence of autoregressive conditional heteroscedasticity (ARCH) effects presented Table 14. The squared disturbance term of equation 1 has an almost constant mean value while its variance is autocorrelated and changes over time. The null hypothesis that all 5 lags of the squared residuals have coefficient values that are not significantly different from zero is rejected at 5 percent level. Again, the decision is based on value of the test statistic computed as the product of R^2 from the auxiliary regression and the number of observations (T) which is greater than the critical value of the chi-square (χ^2) at 5 degrees of freedom. Also, the results suggest that the conditional correlation between spot and futures prices is also time-varying rather than constant as assumed in the OLS such that a time-varying minimum-variance hedge ratio should satisfy this data. However, considering the purpose of the study, attention is on a constant GARCH hedge ratio obtained as the slope of the futures return again in order to enable comparison with the other three constant hedge ratios.

Table 14: ARCH Effects Test				
	Coefficient	Standard error	t-Statistic	Probability
γ_1	9.16E-06	1.83E-06	5.008501	0.0000
ε^2_{t-1}	0.283038	0.031208	9.069307	0.0000
ε^2_{t-2}	-0.006326	0.032433	-0.195046	0.8454
ε^2_{t-3}	-0.022910	0.032431	-0.706419	0.4801
ε^2_{t-4}	-0.009684	0.032437	-0.298553	0.7653
ε^2_{t-5}	0.013476	0.032437	-0.415444	0.6779
R^2	0.080786			
Included observations	1037			
F-statistic	11.26044	Probability	0.00000	
Obs*R-squared	83.53308	Probability	0.00000	

Auxiliary equation: $\varepsilon^2_t = \gamma_0 + \gamma_1\varepsilon^2_{t-1} + \ldots + \gamma_q\varepsilon^2_{t-q} + \nu_t$

The ECM-GARCH (1,1) is estimated with the conditional mean (ECM) accounting for the error correction term in section 4.3 and 4.4 while the conditional variance (GARCH (1,1)) by Bollerslev (1986) allows for the second moment in the error structure. It is noteworthy that an increase of the lag-length on ΔS_t and ΔF_t in the mean equation beyond 1 result in insignificant coefficient estimates. The GARCH (1, 1) order is valid because the first lags of the ARCH and GARCH terms minimise the SBIC while the AIC is biased for the tenth lag in Table 15.

Table 15: GARCH Lag-length Selection		
Lag length	Akaike	Schwarz
1	- 8.693526	- 8.655502*
2	- 8.677288	- 8.629721
3	- 8.706169	- 8.649045
4	- 8.703949	- 8.637252
5	- 8.699482	- 8.623199
6	- 8.707606	- 8.621722
7	- 8.707265	- 8.611764
8	- 8.713155	- 8.608023
9	- 8.711958	- 8.597181
10	- 8.715615*	- 8.591176

*Notes: * Lag length that minimises information criterion.*

Empirical results in Table 16 indicate that it took the Marquadt logarithm 63 iterations to maximise the likelihood function and computed robust standard errors. Focus on the conditional variance equation, reveals that the weights of the parameters ($\alpha_1 + \beta + \gamma$) are positive and sum up to less than which is required to have a mean-reverting variance process. Since the sum is very close to one, this process only reverts very slowly. Both the lagged squared residuals (ε^2_{t-1}) and lagged conditional variance (σ^2_{t-1}) are significantly different from zero at 5 percent level and this attests to theory that the GARCH (1,1) error is able to capture the dynamics in the second moments of the conditional distribution of returns. Additionally, the unconditional variance derived from the sum of β and γ is constant because ($\beta + \gamma < 1$). Effectively, the results show that that the GARCH model is mean-reverting and conditionally heteroscedastic, but has a constant unconditional variance.

Table 16: GARCH (1,1)

$$100\,\Delta S_t = \alpha_0 + \theta\Delta S_{t-1} + h_{garch}\Delta F_t + \lambda\Delta F_{t-1} + \psi\hat{u}_{t-1} + \varepsilon_t$$
$$\varepsilon_t \mid \Omega_{t-1} \sim N(0, \sigma^2_t)$$
$$\sigma^2_t = \alpha_1 + \beta\varepsilon^2_{t-1} + \gamma\sigma^2_{t-1}$$

	Coefficient	Standard Error	z-statistic	Probability
α_0	0.000171	8.09E-05	2.109551	0.0349
ΔS_{t-1}	-0.253586	0.024331	-10.42239	0.0000*
ΔF_t	0.922332	0.007540	122.3321	0.0000*
ΔF_{t-1}	0.243620	0.023518	10.35909	0.0000*
$\hat{u}_{t-1}$	-0.189835	0.016063	-11.81850	0.0000*
α_1	5.52E-09	7.23E-09	0.763868	0.4449
ε^2_{t-1}	0.016373	0.002681	6.108039	0.0000*
σ^2_{t-1}	0.983232	0.002195	447.9092	0.0000*

Log likehood = 4532.981

Notes: convergence achieved after 63 iterations.
** indicates significance level at 5 percent.*

All statistical conditions of the model are satisfied, as the estimated parameters including the hedge ratio in the conditional mean equation are also significantly different from zero at 5 percent level being consistent with results attained in section 4.3. Therefore, emphasis is now on the hedge ratio ($h_{garch} = 0.9223$) which is the largest in value relative to the other hedge ratios in Table 17. From the empirical analysis it is suggested that the ECM-GARCH (1,1) can result in the optimal risk minimisation for a spot position in the South African stock exchange market. This is because the model corrects for the inefficiency of the OLS hedge ratio as well as the limitation of ECM and VECM hedge ratios with respect to the time-varying conditional heteroscedasticity observed in the returns.

Table 17: Estimated Hedge Ratios			
OLS	ECM	VECM	ECM-GARCH(1,1)
0.9057	0.9199	0.9199	0.9223

The OLS, ECM and VECM seem to underestimate the spot position to be hedged in futures contracts. Sephton (1993) also obtains a large hedge ratio from a univariate GARCH model relative to the OLS for wheat futures. In addition, Floros et al. (2004) obtain a slightly large mean hedge ratio from a bivariate GARCH (1,1) and conclude that the model is superior to the OLS, ECM and VECM and should be more efficient in reducing the risk of the spot price.

Chapter 5

Conclusion

The essential factor in hedging with stock index futures is estimating the optimal hedge ratio. In this study, four econometric models are specified to estimate hedge ratios and determine which model potentially offers the minimum risk of spot prices for a given portfolio of assets. Investigation of the statistical characteristics of the FTSE/JSE Top 40 stock index and the corresponding index futures in the South African Futures Exchange Market (SAFEX) facilitate the choice of model specification.

First, the Ordinary Least Square (OLS) regression adopted as a traditional technique and also for the wide application in risk management generates the smallest and statistically inefficient constant hedge ratio. This hedge ratio under-estimates the number of futures contracts required by an investor to hedge spot price risk hence all other things remaining equal, a risk-averse investor should avoid the OLS technique. The outcome is a consequence of the violation of assumptions underlying the classical linear regression model because of the time-varying distribution and dependency that are common features of stock index and index futures prices and respective returns.

Since the integrated spot and futures prices exhibit a long-term cointegration relationship, such cointegration justifies recourse to the second model with an error correction term (ECM). In this case, the hedge ratio is slightly large in value and efficient compared with the OLS, as stipulated in theory and early research. The evidence shows that an investor requires slightly more futures contracts to hedge the risk exposure in the spot market. The third model, a vector error correction model (VECM) restricts the exact number of cointegrating equations although it fails to offer a unique hedge ratio in this study. Given that the estimated hedge ratio is the same as the one derived from the ECM, an investor would rather avoid this lengthy hedging process in favour of the ECM when given the three options only. However, empirical results may change with a different sample size.

In the last model conditional heteroscedasticity that has been well established as one of the key features of financial time series is taken into account alongside the cointegration relationship between the spot and futures. The ECM-GARCH (1,1) provides the largest hedge ratio in value and this reflects the statistical shortfalls of the previous hedge ratios as they can result in hedging a spot portfolio that is less than required to reduce risk. Therefore, the GARCH model should is potentially efficient and superior in reducing the risk of the spot price. However, the time- path of the GARCH hedge ratio is also restricted in this model hence there is no recommendation for a re-adjustment to meet changing market conditions.

The study will be extended in future to evaluate the hedging performance of the OLS, ECM, VECM and ECM-GARCH (1,1) hedge ratios measured in terms of ex-ante and ex-post risk-return trade-off at various forecasting horizons. In addition, a bivariate GARCH model will be adopted in order to estimate a time-varying hedge ratio. Empirical evidence based on the extension will provide an answer to whether the GARCH model offers significant portfolio risk reduction relative to the other models in the South African futures exchange market.

References

Anderson, R. W. and J. Danthine, 1981. Cross hedging [Electronic Version]. *The Journal of Political Economy,* 89(6), 1182-1196.

Baillie, R. T. and R. J. Myers, 1991. Bivariate GARCH estimation of the optimal commodity futures hedge [Electronic Version]. *Journal of Applied Econometrics*, 6(2), 109-124.

Bollerslev, T., 1986. Generalised autoregressive conditional heteroscedasticity [Electronic Version]. *Journal of Econometrics,* 31(3), 307-327.

Brooks, C., 2002. *Introductory econometrics for finance*. Cambridge: Cambridge University Press.

Brooks, C., Henry, O. T., and G. Persand, 2002. The effect of asymmetries on optimal hedging [Electronic Version]. *Journal of Business,* 75(2), 333-352.

Castelino, M. G., J. C. Francis and A. W. Wolf, 1991. Cross-hedging: basis risk and choice of the optimal hedging vehicle [Electronic Version]. *The Financial Review,* 26(2), 179-210.

Cecchetti, S. G., R. E. Cumby and S. Figlewski, 1988. Estimation of the optimal futures hedge [Electronic Version]. *Review of Economics and Statistics,* 70(4), 623-630.

Chen, S., C-f.Lee and K. Shrestha, 2003. Futures hedge ratios: a review [Electronic Version]. *The Quarterly Review of Economics and Finance,* 43(3), 433-465.

Chou, W. L., K.K.F. Denis and C. F. Lee, 1996. Hedging with the Nikkei index futures: the conventional model versus the error correction model [Electronic Version]. *The Quarterly Review of Economics and Finance,* 36(4), 495-505.

Ederington, L. H., 1979. The hedging performance of the new futures markets [Electronic Version]. *The Journal of Finance,* 34(1), 157-170.

Engle, R. F., 1982. Autoregressive conditional heteroscedasticity with estimates of variance of United Kingdom inflation [Electronic Version]. *Econometrica,* 50(4), 987-1008.

Engle, R. F. and C. W. Granger, 1987. Cointegration and error correction: representation, estimation and testing [Electronic Version]. *Econometrica,* 55(2), 251-276.

Figlewski, S., (1984). Hedging performance and basis risk in stock index futures futures [Electronic Version]. *The Journal of Finance,* 39(3), 657-669.

Floros, C. and D. V. Vougas, (2004). Hedge ratios in Greek stock index futures market [Electronic Version]. *Applied Financial Economics*, 14(15), 1125-1136.

FTSE/JSE Top 40 Futures Prices. Retrieved in various days, February 2006 from the South African Futures Exchange, Johannesburg Stock Exchange Web site: http://www.safex.co.za/

Ghosh, A., 1993. Hedging with stock index futures: estimation and forecasting with error correction model. *The Journal of Futures Markets,* 13 (7), 743-752.

Gujarati, D. N., 2003. *Basic econometrics* (4th ed.). New York: McGraw-Hill.

Hull, J. C., 1997. *Options, futures and other derivatives* (3rd ed.). London: Prentice Hall.

Johnson, L. L., 1960. The theory of hedging and speculation in commodity futures [Electronic Version]. *The Review of Economics Studies,* 23(10), 139-151.

Lien, D. H., 1996. The effect of the cointegration relationship on futures hedging: a note. *The Journal of Futures Markets,* 16(7), 773-780.

Lien, D. H. and X. Luo, 1993. Estimating multiperiod hedge ratios in cointegrated markets. *Journal of Futures*, 13(8), 909-920.

Lien, D. H. and X. Luo, 1994. Multi-period Hedging in the Presence of conditional Heteroscedasticity [Electronic Version]. *The Journal of Futures Markets,* 14(8), 927-955.

Lien, D. H. and Y. K. Tse, 1999. Fractional cointegration and futures hedging. *The Journal of Futures Markets,* 19(4), 457-474.

Lien, D., Y. K. Tse and A. K. C. Tsui, 2002. Evaluating the hedging performance of the constant correlation GARCH model [Electronic Version]. *Applied Financial Economics,* 12(11), 791-798.

Lien, D., 2004. Co-integration and the optimal hedge ratio: the general case [Electronic Version]. *The Quartely Review of Economics and Finance,* 44(5), 654-658.

Malliurus, A. G. and J. Urrutia, 1991. Tests of random walk of hedge ratios and measures of hedging effectiveness for stock indexes and foreign currencies. *The Journal of Futures Markets,* 16(7), 773-780.

Myers, R. J. and S. R. Thompson, 1989. Generalized optimal hedge ratio estimation [Electronic Version]. *American Journal of Agricultural Economics,* 71(4), 858-867.

Myers, R. J., 1991. Estimating time-varying hedge ratios on futures markets. *The Journal of Futures Markets,* 11(1), 39-53.

Officer, R. R., 1972. The distribution of stock returns [Electronic Version]. *Journal of American Statistical Association,* 67(340), 807-812.

Pagan, A., 1996. The econometrics of financial markets [Electronic Version]. *Journal of Empirical Finance,* 3(1), 15-102.

Park, T. H. and L. N. Switzer, 1995. Time-varying distributions and the optimal hedge ratios for stock index futures [Electronic Version]. *Applied Financial Economics,* 5(3), 131-137.

Sephton, P. S., 1993. Optimal hedge ratios at the Winnipeg commodity exchange [Electronic Version]. *The Canadian Journal of Economics,* 26(1), 175-19.

Stoll, H. R. and R. E. Whaley, 1990. The dynamics of stock index and stock index futures returns [Electronic Version]. *The Journal of Financial and Quantitative Analysis,* 25(4), 441-468.

Sutcliffe, C. M. S., 1997. *Stock index futures* (2nd ed.). London: International Thompson Business Press.

Wilkinson, K. J., L. C. Rose and M. R. Young, 1999. Comparing the effectiveness of traditional and time varying hedge ratios using New Zealand and Australian debt futures contracts [Electronic Version]. *The Financial Review,* 34(3), 79-94.

Working, H., 1953. Futures trading and hedging, [Electronic Version]. *American Economic Review,* 43(3), 314-343.

Yang, W., 2001. M-GARCH hedge ratios and hedging effectiveness in Australian futures markets. Retrieved June 28, 2006, from Edith Cowan University, School of Finance and Business Economics, Web site: http://www.business.ecu.edu.au/ schools/ afe/fimarc/working_papers.htm