

Should the SARB have Stayed Time Inconsistent?

Rangan Gupta

Associate Professor

University of Pretoria

and

Josine Uwilingiye

Ph.D. Student

University of Pretoria

Objectives:

To answer: The Question Posed in the Title – We must first ask: Whether the SARB was Time Inconsistent in the Pre-Inflation Targeting Period?

If yes, then the question is: Whether the Economy would have done better (**Lower Average Levels of Inflation**), if the SARB continued to be Time Inconsistent instead of moving into an Inflation-Targeting Framework?

We do these by testing the long- and short-run restrictions implied by a *modified* version of the Barro-Gordon's (1983) model over 1960:01-1999:04, and, then, forecasting one-step ahead inflation over 2000:01-2007:04.

Motivation (Inflation hasn't ruined everything. A dime can still be used as a screwdriver):

- Evaluation of the Inflation Targeting Regime – the 3 percent-6 percent band (Welfare Costs);
- More importantly, for this paper – Issue is not comparing inflation levels pre- and post (till date) of the regime, but what if the old regime continued (Starting Point of a Bigger Project (Cepstrum));
- Narrower (Lower) Target Band (Demertzis and Viegi (2005, 2006, 2007, and 2008)).

The Modified Barro-Gordon (1983) Model:

(i) The Lucas-Supply Curve:

$$y_t = y_t^n + \alpha(\pi_t - \pi_t^e); \quad \alpha > 0$$

(ii) The Change in the Natural Rate follows an AR(1) Process:

$$y_t^n - y_{t-1}^n = \lambda(y_{t-1}^n - y_{t-2}^n) + \epsilon_t; \quad 1 > \lambda \geq 0; \quad \epsilon_t \rightarrow iid N(0, \sigma_\epsilon^2)$$

(iii) Beginning of each $t = 0, 1, 2, \dots$, after private agents form π_t^e but prior to the realization of ϵ_t , monetary authority chooses π_t^p (planned-inflation), but with control error $\eta_t \rightarrow iid N(0, \sigma_\eta^2)$, with $\sigma_{\epsilon\eta}$ as covariance with ϵ_t . Actual Inflation then is:

$$\pi_t = \pi_t^p + \eta_t$$

The Modified Barro-Gordon (1983) Model (Continued):

(iv) The Monetary Authority Chooses π_t^p in order to minimize a loss function that penalizes variations of output and inflation around target values $ky_t^n > (y_t^n)$ and 0:

$$L_t = \frac{1}{2}(y_t^n - ky_t^n)^2 + \frac{b}{2}\pi_t^2; \quad b > 0$$

Using (i) -(iii), Monetary Authority's Problem can be re-written as:

$$\min_{\pi_t^p} E_{t-1} \left[\frac{1}{2} \left((1 - k)y_t^n + \alpha(\pi_t^p - \pi_t^e + \eta_t) \right)^2 + \frac{b}{2} (\pi_t^p + \eta_t)^2 \right]$$

Note private agents know the true structure of the economy and understand the monetary authority's time-inconsistency problem. In equilibrium, therefore, $\pi_t^p = \pi_t^e$. Moreover $E_{t-1}\eta_t = 0$.

Solution and Long-Run Implications:

$$\pi_t = \alpha A y_{t-1}^n + \alpha A \lambda \Delta y_{t-1}^n + \eta_t \quad (1)$$

$$y_t = y_{t-1}^n + \lambda \Delta y_{t-1}^n + \epsilon_t + \alpha \eta_t \quad (2)$$

(1) and (2) \Rightarrow that π_t and y_t are non-stationary variables, inheriting the unit root from the underlying process for y_t^n .

Combining (1) and (2):

$$\pi_t - \alpha A y_t = -\alpha A \epsilon_t + (1 - \alpha^2 A) \eta_t \quad (3)$$

(3) \Rightarrow the linear combination $\pi_t - \alpha A y_t$ is stationary, i.e., π_t and y_t are cointegrated, with $A = \frac{k-1}{b} > 0$.

Short-run Implications:

(i)-(iii) $\Rightarrow y_t = y_t^n + \alpha\eta_t$. Taking first-differences yields:

$$\Delta y_t = \Delta y_t^n + \alpha\eta_t - \alpha\eta_{t-1} \quad (4)$$

Combining (3) and (4), and simplifying:

$$\Delta\pi_t = \alpha\lambda\Delta y_t^n + \eta_t - \eta_{t-1} - \alpha A\epsilon_t + \alpha A\epsilon_{t-1} \quad (5)$$

(5) can have a State-Space representation:

$$\xi_t = F\xi_{t-1} + Qv_t \quad (6)$$

$$z_t = H\xi_t \quad (7)$$

Short-run Implications (Continued):

$$\text{We have: } \xi_t = \begin{bmatrix} \Delta y_t^n \\ \epsilon_t \\ \epsilon_{t-1} \\ \eta_t \\ \eta_{t-1} \end{bmatrix}; F = \begin{bmatrix} \lambda & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}; Q = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix};$$

$$H = \begin{bmatrix} \alpha A & -\alpha A & \alpha A & 1 & -1 \end{bmatrix}; z_t = [\Delta \pi_t]; v_t = \begin{bmatrix} \epsilon_t \\ \eta_t \end{bmatrix}, \text{ with } E(v_t v_t')$$

$$= \begin{bmatrix} \sigma_\epsilon^2 & \sigma_{\epsilon\eta} \\ \sigma_{\epsilon\eta} & \sigma_\eta^2 \end{bmatrix}.$$

Note (5) can be re-written as :

$$\Delta \pi_t = \lambda \Delta \pi_{t-1} + \eta_t - (1 + \lambda) \eta_{t-1} + \lambda \eta_{t-2} + (1 + \lambda) \alpha A \epsilon_{t-1} - \alpha \lambda A \epsilon_{t-2} \quad (8)$$

Short-Run Restrictions:

The unrestricted-version of (8) is an ARMA(1, 2) model of $\Delta\pi_t$:

$$\Delta\pi_t = \phi_1\Delta\pi_{t-1} + \phi_2\epsilon_{t-1}^\pi + \phi_3\epsilon_{t-2}^\pi + \phi_4\epsilon_{t-1}^y + \phi_5\epsilon_{t-2}^y + \epsilon_t^\pi \quad (9)$$

One can test for the short-run implications using a likelihood ratio statistic: $2(L^u - L^c)$ which follows a χ^2 with 2 degrees of freedom.

Estimation:

Data: Quarterly Data: 1960:01-2007:04 from SARB on CPI Inflation and Real GDP;

Using ADF, PP, DF-GLS, KPSS tests: **Inflation and Real GDP are non-stationary;**

Long-Run Implications: Using Phillips-Ouliaris (1990) approach (and Johansen cointegration tests- Trace and Eigen Value): One cointegrating relationship of the form $\pi_t - 1.6119y_t = 0$, with no serial correlation in the error term.

Estimation (Continued):

Short-Run Estimates: $\alpha = 1.2716^{***}$ (Burger and Marinkov (2006)); $A = 1.2714^*$ (Gupta and Naraidoo (2008)); $\lambda = 0.33202$; $\sigma_\epsilon = 0.37127$; $\sigma_\eta = 2.438^{***}$; $\sigma_{\epsilon\eta} = -0.90514^*$;

Short-Run Restrictions: $2(562.028-557.604) = 8.848 < 9.21$ (99 percent critical value for a χ^2 with 2 degrees of freedom) \Rightarrow the model's implications cannot be rejected at the 1 percent level of significance.

Conclusions:

- Discretion would have produced on average lower levels of inflation (1.20 or 0.41 percent) than the current regime;
- Three-Equation Model: Too Simplistic? How Reliable are the Results? (Short-Run Dynamics Need More Work (Perhaps)!);
- Policy Implication \Rightarrow Managing the Inflation Targeting Framework Better \Rightarrow Narrower (Lower) Target Band (Demertzis and Viegli (2005, 2006, 2007, and 2008)).